Reg.No.										

MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institute of Manipal University, Manipal

I SEM M. Tech. (CAAD) DEGREE END SEMESTER MAKEUP EXAMINATIONS DECEMBER 2016

SUBJECT: SOLID MECHANICS (MME 5101)
REVISED CREDIT SYSTEM

Time: 3 Hours. Max. Marks: 50

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data, if any, may be assumed appropriately.

1. a) Derive the equations of equilibrium in cylindrical coordinates **(06)**

b) The displacement field is given by, $u = kxy, v = kxy \; and \; w = 2k(x + y)z$, where *u, v* and *w* are the displacements in x, y and z directions respectively and k is a constant small enough to ensure applicability of the small deformation theory. Evaluate, **(04)**

i) The strain matrix/state

ii) The strain in the direction, $n_x = n_y = n_z = \frac{1}{\sqrt{3}}$

2. a) State and discuss the Elastic Energy theory of failure and obtain the equation for evaluating elastic energy in a body subjected to three dimensional state of stress. **(05)**

b) A cylindrical rod is subjected to a torque *T*. At any point *P* of the cross section, the following stresses occur **(05)**

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0; \; \tau_{yz} = G\theta x; \; \tau_{zx} = -G\theta y$$

Check whether these satisfy the equations of equilibrium. Also show that the lateral surface is free of load, i.e show that $T_x = T_y = T_z = 0$.

3. a) Derive the cubic equation which gives the state of principal strain at a point in the body in the form, **(05)**

$$\epsilon^3 - J_1 \epsilon^2 + J_2 \varepsilon - J_3 = 0$$

Where, J_1, J_2 and J_3 are the strain invariants.

b) Determine the principal stresses and their directions for the state of stress characterized by the following stress matrix: **(05)**

$$[\sigma_{ij}] = \begin{bmatrix} 3 & -10 & 0 \\ -10 & 0 & 30 \\ 0 & 30 & -27 \end{bmatrix}$$

Note: All stress components are in MPa.

4. a) The displacement field for a body is given by

$$u = [\,(x^2 + y^2 + 2)i + (3x + 4y^2)j + (2x^3 + 4z)k\,] \times 10^{-4}$$

i) What is strain at P(1,2,3) in the direction of ***PQ*** having direction cosines $n_x = 0.6$, $n_y = 0$ and $n_z = 0.8$?

ii) What is the orientation of ***P'Q'*** after deformation? **(06)**

b) Verify whether the following strain field satisfies the equations of compatibility, if p is a constant. **(04)**

$$\varepsilon_{xx} = py,\ \varepsilon_{yy} = px,\ \varepsilon_{zz} = 2p(x+y)$$

$$\gamma_{xy} = p(x+y),\ \gamma_{yz} = 2pz \text{ and } \gamma_{xz} = 2pz$$

5. a) What is the sufficient condition for pure shear state at a point? Discuss the significance of decomposing a general stress state at a point into hydrostatic and pure shear states? **(04)**

b) Obtain the stress-strain relations for linear isotropic materials obeying Hooke's law in the following form: **(06)**

$$\varepsilon_{xx} = \frac{1}{E}[\sigma_x - \vartheta(\sigma_y + \sigma_z)]$$

$$\varepsilon_{yy} = \frac{1}{E}[\sigma_y - \vartheta(\sigma_x + \sigma_z)]$$

$$\varepsilon_{zz} = \frac{1}{E}[\sigma_z - \vartheta(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G};\quad \gamma_{yz} = \frac{\tau_{yz}}{G} \quad \textit{and}\ \gamma_{xz} = \frac{\tau_{xz}}{G}$$

Where ε_{xx}, ε_{yy} and ε_{zz} are linear strains in ***x, y*** and ***z*** directions respectively γ_{xy}, γ_{yz} and γ_{xz} are shear strains in ***xy, yz*** and ***xz*** planes respectively.