Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

I SEMESTER M.TECH (THERMAL SCIENCES AND ENERGY SYSTEMS)

END SEMESTER MAKE-UP EXAMINATIONS DEC 2016/JAN 2017

SUBJECT: FEM FOR THERMAL ENGINEERING [MME 5142]

REVISED CREDIT SYSTEM
(29/12/2016)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1 A) Derive the shape functions for a uni-dimensional heat conducting Isoparametric bar element of length L, having three equally spaced nodes using Lagrangian Interpolation formula. Deduce the thermal conductance matrix for the same having thermal conductivity, K_x and constant area of cross-section A_x. Assume that there is neither heat generation nor convection applied on the element, but subjected to end heat fluxes only. (06)

1 B) Determine the temperature of the plate element shown in **Fig. 1** below, at point P. The nodal temperatures are given as T_1= 80 ^oC, T_2= 25 ^oC and T_3=40 ^oC. (04)

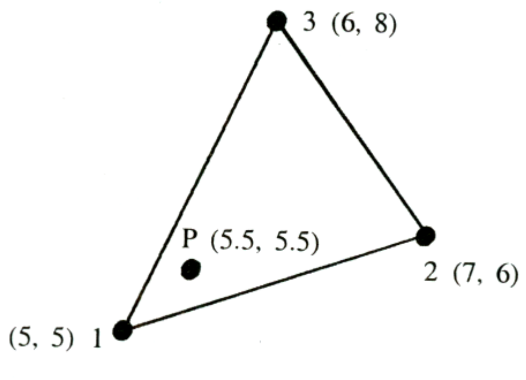

Fig. 1

2 A) For **Unsteady state one dimensional heat conduction** in a bar subjected to uniform heat generation G W/m^3, and convection over its lateral surface and subjected to end heat fluxes, derive with usual notation , the following thermal equilibrium finite element equation, (07)

$$\frac{\rho c_p l A}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\dfrac{\partial T_i}{\partial t}\\ \dfrac{\partial T_j}{\partial t}\end{Bmatrix}+\left(\frac{Ak_x}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\frac{hPl}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{Bmatrix}T_i\\ T_j\end{Bmatrix}$$

$$=\frac{GAl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}-\frac{qPl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}+\frac{hT_aPl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

2 B) Determine the Jacobian matrix for a four noded two dimensional quadrilateral element described by its Isoparametric parent element. (03)

3A) For the composite plane having wall one dimensional steady state heat transfer as shown in **Fig.2**, obtain the finite element equilibrium equations in the form. $[K]_{th}\{T\}=\{Q\}_{loads}$ (04)

Use discrete system analysis using energy balance at each node.

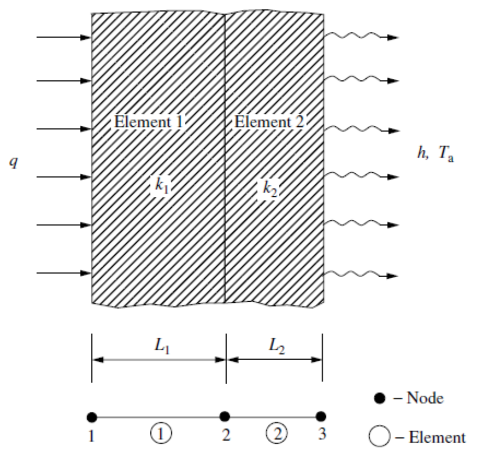

Fig. 2

3 B) Use the Galerkin weighted residual formulation to obtain the Thermal Conductance and Load matrices for a slender one dimensional fin with the open end insulated. Assume a two noded linear steady state heat transfer element. The Governing Differential Equation (with usual notations), is given by, (06)

$$K\frac{d^2T}{dx^2}-\left(\frac{P}{A}\right)h(T-T_\infty)=0$$

4 A) Determine the nodal Conductance Matrix and Thermal Load vector for a three noded linear triangular thermal element having K = 20W/mm·K and thickness 10 mm as given in **Fig 3** below: (07)

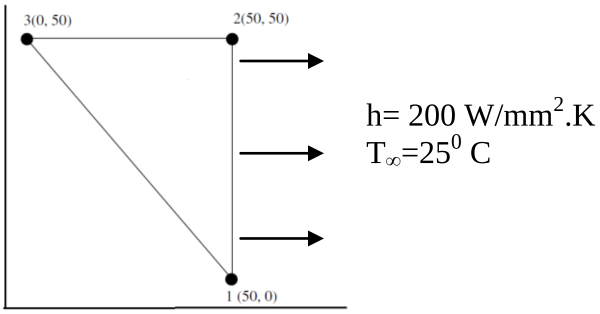

Fig. 3

4 B) Give an example for each to illustrate the use of Eular- Lagrange Equations to deduce the Governing Differential Equations for any three physical systems. (03)

5 A) Explain what is meant by Area Coordinates. What is their use? (03)

5 B) Apply Galerkin's weighted residual formulations to obtain the stiffness conductance analogous matrix as well as the thermal load matrix for an **axisymmetric triangular thermal element** in the form as given below. (07)

$$[\mathbf{K}] = \frac{2\pi\bar{r}k_r}{4A}\begin{bmatrix} b_i^2 & b_ib_j & b_ib_k \\ b_ib_j & b_j^2 & b_jb_k \\ b_ib_k & b_jb_k & b_k^2 \end{bmatrix} + \frac{2\pi\bar{r}k_z}{4A}\begin{bmatrix} c_i^2 & c_ic_j & c_ic_k \\ c_ic_j & c_j^2 & c_jc_k \\ c_ic_k & c_jc_k & c_k^2 \end{bmatrix}$$

$$+ \frac{2\pi h l_{ij}}{12}\begin{bmatrix} 3r_i + r_j & r_i + r_j & 0.0 \\ r_i + r_j & r_i + 3r_j & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\{\mathbf{f}\} = \frac{2\pi GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{Bmatrix} r_i \\ r_j \\ r_k \end{Bmatrix} - \frac{2\pi q l_{jk}}{6}\begin{Bmatrix} 0 \\ 2r_j + r_k \\ r_j + 2r_k \end{Bmatrix} + \frac{2\pi h T_a l_{ij}}{6}\begin{Bmatrix} 2r_i + r_j \\ r_i + 2r_j \\ 0 \end{Bmatrix}$$