Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institute of Manipal University, Manipal

I SEMESTER M.TECH (THERMAL SCIENCES & ENERGY SYSTEMS)

END SEMESTER EXAMINATION – NOV./DEC. 2016

SUBJECT: FINITE ELMENTS FOR THERMAL ENGINEERING (MME - 5142)

REVISED CREDIT SYSTEM

Time: 3 Hour **Max. Marks: 50**

Note: (i) Answer All Questions
(ii) Missing data, if any, may be appropriately assumed
(iii) Assumptions made must be clearly mentioned

1A. Define an Isoparametric Element. Bring out the salient differences between Sub-parametric and Super- Parametric elements. Compare the relative importance. 03

1B. Derive the **Conductance Matrix** terms for the Axi-Symmetric heat transfer by applying **Galerkin's weighted residual formulation**. The governing equation for axi-symmetric steady state heat transfer is given by (with usual notations) 07

$$k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)+Q=0$$

2A. Use Lagrangian Interpolation formula to determine all the Shape Functions for a Triangular area element with mid-side nodes 03

2B. Solve for nodal temperatures for a quadratic element of K= 50 W/m.K and having uniform cross section of diameter 50 mm and length 0.5 m. The steady state heat flux into the element is 4 kW/m^2. 07

$$k_x A\int_0^L \frac{dN_i}{dx}\left[\frac{dN_1}{dx}T_1+\frac{dN_2}{dx}T_2+\frac{dN_3}{dx}T_3\right]dx = \underline{-k_x A[N_i]\frac{dT}{dx}}\Big|_o^L \quad for\ i=1,2,3$$

3A. Determine the Shape functions for the four noded Isoparametric rectangular element. Compute the temperature at a location (2,1) in the element as shown in Fig.1 03

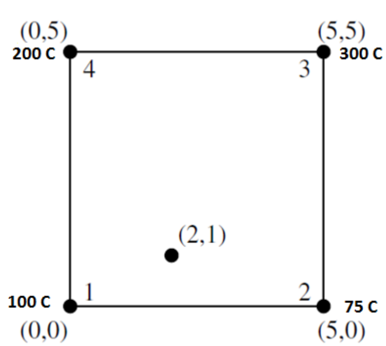

Fig. 1

3B. For the simplified uniaxial two noded fin element **Fig.2** (having length L, perimeter, P across a cross sectional area A), using discrete system analysis, show (with usual notations) that, 07

$$\begin{bmatrix} \frac{kA}{L}+\frac{hPL}{4} & -\frac{kA}{L}+\frac{hPL}{4} \\ -\frac{kA}{L}+\frac{hPL}{4} & \frac{kA}{L}+\frac{hPL}{4} \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix} = \begin{Bmatrix} Q_i+\frac{hPL}{2}T_{\mathrm{a}} \\ -Q_j+\frac{hPL}{2}T_{\mathrm{a}} \end{Bmatrix}$$

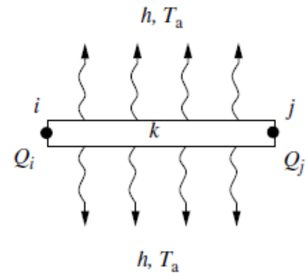

Fig. 2

4A. Use the Variational formulation to obtain the Thermal Conductance and Load matrices for a slender fin. Assume a two noded linear steady state heat transfer element. The Governing Differential Equation (with usual notations), is given by, 07

$$K\frac{d^2T}{dx^2}-\left(\frac{P}{A}\right)h(T-T_\infty)=0$$

4B. Sketch and explain what is meant by Area Coordinates 03

5A. Derive the following General Cartesian Three-Dimensional Finite Element Formulation without Convection by applying Galerkin Weighted Residual Method: 07

$$\left[\iiint\left(k_x\left[\tfrac{\partial N}{\partial x}\right]^T\left[\tfrac{\partial N}{\partial x}\right]+k_y\left[\tfrac{\partial N}{\partial y}\right]^T\left[\tfrac{\partial N}{\partial y}\right]+k_z\left[\tfrac{\partial N}{\partial z}\right]^T\left[\tfrac{\partial N}{\partial z}\right]\right)dV\right]\{T\}=\left\{\iiint\dot{q}_g\,[N]^T\,dV\right\}$$
$$+\left\{\iint\left(\dot{q}_x\hat{n}_x+\dot{q}_x\hat{n}_x+\dot{q}_x\hat{n}_x\right)[N]^T\,dA\right\}$$

5B. Derive the two point sampling Gaussian quadrature formula and evaluate the following integral using the same. Compare the result with the exact analytical solution: 03

$$\int_2^5(2x^3-5x+8)dx$$