Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

FIFTH SEMESTER B.TECH. (INSTRUMENTATION & CONTROL ENGG.)

END SEMESTER EXAMINATIONS, DEC 2016/JAN 2017

SUBJECT: MODERN CONTROL THEORY [ICE 3101]

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A. Derive the state model of a RLC parallel circuit excited by a current source i(t), considering the capacitor voltage $v_c(t)$ as the output variable. Also derive the transfer function $\frac{V_c(s)}{I(s)}$ from the state model. **5**

1B. Consider a dynamic matrix governed by $\dot{x}(t) = Ax + Bu$ where **3**

A= $\begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$.

Determine i) Eigen values ii) Eigen vectors iii) Transformation matrix P iv) Jordan canonical form of A.

1C. A system is described by the transfer function **2**

G(s)= $\frac{Y(s)}{U(s)} = \frac{s^2+4s+4}{s^3+5s^2+4s}$. Develop a state model in observable canonical form.

2A. Obtain the time response of the system **5**

$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$;where u(t) is a unit step input occurring at t=0 and $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

2B. Determine the complete state controllability and observability property of the system **3**

described by $\dot{x}(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}x(t)+\begin{bmatrix}0\\0\\1\end{bmatrix}u(t)$ and y(t)= $[1 \quad 1 \quad 0]\underline{x}$(t).

2C. Illustrate the duality property of observer gain computation. **2**

3A. Consider a system $\dot{x}(t)=Ax(t)+Bu(t);$ y(t)=Cx(t) with **5**

A=$\begin{bmatrix}0 & 10\\ 1 & 10\end{bmatrix}$; B=$\begin{bmatrix}0\\1\end{bmatrix}$ and C= $[0 \quad 1]$. Design a full observer to implement the state feedback u(t)=-kx(t), with eigen values of the observer to be at -8 and -12. With given (A,C) are in observable canonical form, solve using direct comparison method.

3B. A linear discrete time system has the transfer function **3**

$\frac{Y}{U}(z)=\frac{2z+5}{6z^3-5z^2+z}$. Obtain a diagonal realization for the same. Draw the corresponding state diagram.

3C. Find the state space representation for the difference equation y(k+3)+6y(k+2)+11y(k+1)+8y(k)=10u(k). Draw the corresponding state diagram. **2**

4A. Obtain a discrete time state space representation of the following continuous time system $\dot{x}(t)=Ax(t)+Bu(t);$ A=$\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix}$; B=$\begin{bmatrix}0\\1\end{bmatrix}$, with sampling time T=1s. **5**

4B. Given the system matrix **3**

F=$\begin{bmatrix}0 & 1\\ -0.16 & -1\end{bmatrix}$ of a discrete time system. Determine its discrete time state transition matrix using Cayley-Hamilton Theorem.

4C. Comment on the stability and system response of a discrete time pulse transfer function based on location of its poles in the z-plane. **2**

5A. Consider the system x(k+1)=Fx(k)+gu(k); y(k)=cx(k) **5**

Where F=$\begin{bmatrix}0.16 & 2.16\\ -0.16 & -1.16\end{bmatrix}$; g=$\begin{bmatrix}-1\\1\end{bmatrix}$; c= $[1 \quad 1]$.

Using Ackerman's formula design a state feedback control algorithm which gives closed loop characteristic roots at $z_{1,2}$ =0.6±j0.4.

5B. Prove the sufficient condition for Lyapunov stability of an autonomous linear continuous time system described by state model. **3**

5C. Determine whether or not the following quadratic form is positive definite. **2**

i) Q=$x_1^2+4x_2^2+x_3^2+2x_1x_2-6x_2x_3-2x_1x_3$.

ii) Q=$-x_1^2-3x_2^2-11x_3^2+2x_1x_2-4x_2x_3-2x_1x_3$.