Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
Manipal University

SEVENTH SEMESTER B.TECH (E & C) DEGREE END SEMESTER EXAMINATION - NOV/DEC 2016
SUBJECT: INFORMATION THEORY AND CODING (ECE - 407)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates
- Answer **ANY FIVE** full questions.
- Missing data may be suitably assumed.

1A. Let S be zero memory source with source alphabet $S = \{s_i\}, i = 1,2,\dots,q$, and symbol probabilities $P_1, P_2, \dots, P_q$. Construct a new zero memory source S' with twice as many symbols, $S' = \{s_i'\},\ i = 1,2,\dots,2q$. Let P_i', the symbol probabilities for the new source, defined by $P_i' = (1-\epsilon)P_i,\ for\ \ i = 1,2,\dots,q$ and $P_i' = \epsilon P_{i-q}\quad i = q+1, q+2,\dots,2q$. Express $H(S')$ in terms of $H(S)$.

1B. A dice is thrown twice. Find P (A/B) if (a) A, a unity occurs on the first throw, B, the score is less than four, (b) A, a five occurs on the second throw, B, the score is not less than 10, (c) A, even occurs on the second throw, B, odd occurs on the first throw.

1C. Starting from the logarithmic inequality, show that the entropy of the discrete memoryless source is maximum if its symbols are equiprobable.

(5+3+2)

2A. The state diagram of a first order binary Markov source S is given in Figure Q2A. Compute H(S) and H($\bar{S}^2$) . Identify extensions S^2 and S^3 and hence compute, $H(\overline{S^2})$ and $H(\overline{S^3})$.

2B. For a discrete memoryless source S with alphabet S={s_i},i=1,2,…q, and probabilities {P_i}, i=1,2,…,q, prove that $H(S^n) = nH(S)$.

2C. Define and explain ergodic and nonergodic Markov source.

(5+3+2)

3A. An information source produces sequences of independent symbols having the following probabilities.

A	B	C	D	E	F	G
1/3	1/27	1/3	1/9	1/9	1/27	1/27

(i) Using Shannon Fano procedure, construct a binary code and determine the efficiency and redundancy of the code.

(ii) For the same source, construct binary and trinary codes using Huffman procedure. Compute efficiency and redundancy in each case.

3B. A source has 6 possible outputs with probabilities as given in the table. Codes A, B, C, D, E and F are considered.

(i) Which of these are uniquely decodable?
(ii) Which are instantaneous
(iii) Determine the average length for all uniquely decodable codes.

Output(si)	**P(si)**	**A**	**B**	**C**	**D**	**E**	**F**
S1	1/2	000	0	0	0	0	0
S2	1/4	001	01	10	10	10	100
S3	1/4	010	011	110	110	1100	101
S4	1/16	011	0111	1110	1110	1101	110
S5	1/16	100	01111	11110	1011	1110	111
S6	1/16	101	011111	111110	1101	1111	001

3C. A source S has six symbols with probabilities P1 to P6, respectively. Assume that we have ordered the Pi so that P1≥P2≥.......≥P6. We wish to find a compact code for this source using the code alphabet X = {0, 1, 2, 3}. Determine a set of word lengths for such a compact code if P6 = 1/64.

(5+3+2)

4A. Encode the message "**MISSISSIP**" generated by a source with 26-letter alphabet (A-Z) using Adaptive Huffman coding.

4B. Consider a binary symmetric channel, whose input is the alphabet X = {0, 1} with probabilities {0.5, 0.5}; whose output alphabet is Y = {0, 1}; and whose channel matrix is

$$\begin{bmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{bmatrix}$$

where ϵ is the probability of transmission error.

a. Compute H(X), H(Y), H(X, Y) and I(X,Y)
b. How many values are there for ϵ for which the mutual information of this channel is maximum? What are those values, and what is the capacity of this channel?

4C. Calculate the capacity of a binary erasure channel assuming *p* as the probability of error. (5+3+2)

5A. Consider a channel with x_1, x_2, x_3 input symbols and y_1, y_2, y_3 output symbols described by the matrix.

$$P(X,Y) = \begin{bmatrix} 3/40 & 1/40 & 1/40 \\ 1/20 & 3/20 & 1/20 \\ 1/8 & 1/8 & 3/8 \end{bmatrix}$$

a. Calculate the entropies of X and Y.
b. Calculate the joint and conditional entropies: H(X,Y), H(X/Y), H(Y/X)
c. Calculate the average mutual information I(X;Y).

5B. For a three variable system, H(Y,Z/X) = 0.43bits/symbol & H(Y,Z)=0.725bits/symbol & H(X)=0.333bits/symbol, compute mutual information of X and (Y,Z) and equivocation of X with respect to Y and Z.

5C. Determine the non-systematic Hamming code for the message stream (1001)

(5+3+2)

6A. Determine the channel capacity of an r-ary symmetric channel for an overall probability of error p. Also compute the rate of transmission for a quaternary symmetric channel if p=1/3 and symbol rate is 100000 symbols/sec.

6B. Define maximum likelihood decision rule. Derive the expression for the probability of error in terms of channel matrix parameters if a priori probabilities are equal.

6C. Compute the probability of error of the channel described by its channel matrix as given below, using maximum likelihood decision rule.

$$\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

(5+3+2)

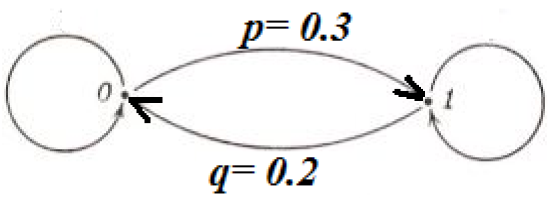

Figure Q2A