Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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SEVENTH SEMESTER B.TECH (E & C) DEGREE END SEMESTER EXAMINATION - NOV/DEC 2016
SUBJECT: INFORMATION THEORY AND CODING (ECE - 407)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates
- Answer **ANY FIVE** full questions.
- Missing data may be suitably assumed.

1A. Two zero-memory sources $S1$ and $S2$ have $q1$ and $q2$ symbols, respectively. The symbols $S1$ occur with probabilities $Pi, i = 1,2,\dots q1$; the symbols of $S2$ occur with the probabilities $Qi, for\ i = 1,2,3,\dots q2$; and the entropies of $S1$ and $S2$ are $H1$ and $H2$, respectively. A new zero memory source $S(\lambda)$, called a mixture of $S1$ and $S2$, is formed with $q1 + q2$ symbols. The first q1 symbols of $S(\lambda)$ have probabilities $\lambda Pi, for\ i = 1,2,3,\dots q1$, and the last $q2$ symbols of $S(\lambda)$ have probabilities $\bar{\lambda} Qi, for\ i = 1,2,3,\dots q2$ $(\bar{\lambda} = 1 - \lambda)$. Express H $[S(\lambda)]$ in terms of $H1, H2\ and\ H(\lambda)$. Express λ_0, the value of λ which maximizes H(S(λ)).

1B. A circuit consists of two components operating independently. The probability that the first component will fail when the circuit is switched on is 0.05 and that of the second is 0.08. Determine the probability that when the circuit is switched on (a) only the first component will fail, (b) both components will fail, (c) only the second component will fail, (d) both components will operate.

1C. Define Entropy. Compute the Entropy of a source S in bits, Nats and Hartleys which has the probability distribution P = {1/4,1/4,1/8,1/8,1/8,1/16,1/16}.

(5+3+2)

2A. The state diagram of a first order binary Markov source S, given in Fig Q2A. Compute H(S). Identify extensions S^2 and S^3 with probabilities and hence compute $H(\bar{S}^2)$, $H(\overline{S^2})$ and $H(\overline{S^3})$.

2B. Let S_0 be the third extension of a zero memory binary source with the probability of a 0 equal to p. Another source, S, observes the output of S_0 and emits either a 0, 1, 2 or 3 according to whether the output of S_0 had 0,1, 2, 3 zeros.Compute $H(S_0)$ and $H(S)$.

2C. Prove that for a given source, S, the minimum possible average length of any uniquely decodable code is equal to its entropy.

(5+3+2)

3A. A discrete memoryless source, S, consists of 3 symbols, x1, x2, x3, with probabilities 0.45, 0.35 and 0.2 respectively.
(i) determine a binary compact code for the source, S
(ii)determine a binary compact code for the second extension of the source
(iii) determine the code efficiencies for the 2 cases.

3B. Consider a source S, with symbols s1, s2,….,s8, having probabilities Pi=$\{\frac{1}{4},\frac{1}{4},\frac{1}{8},\frac{1}{8},\frac{1}{16},\frac{1}{16},\frac{1}{16},\frac{1}{16}\}$. Using Shannon Fano coding, find an instantaneous code for S, when X= {0,1}. Also calculate efficiency of the code.

3C. Consider a source S with source alphabet S={s1,s2,….,s6} having probabilities

Pi=$\{\frac{1}{3},\frac{1}{4},\frac{1}{8},\frac{1}{8},\frac{1}{12},\frac{1}{12}\}$. Construct a Huffman code for the source, S, if the code alphabet, X= {0,1,2,}.

(5+3+2)

4A. A source with 26 alphabets (A-Z) generates a message "**ABRACADA**". Encode this message using Adaptive Huffman coding algorithm.

4B. Two binary symmetric channels are cascaded as shown in Fig. Q 4B. Determine joint entropy H(X, Z) and overall mutual information I(X, Z) given that $p(x_1) = 0.6$ and $p(x_2) = 0.4$.

4C. Show that H(X, Y) = H(X/Y) + H(Y).

(5+3+2)

5A. A channel transmit symbols, a1, a2, a3 with probabilities 0.3, 0.25 and 0.45 respectively. If B is the output symbol alphabet, calculate: H(A), H(B), P(A/B), H(A/B), H(A, B) and I(A; B) if the

Channel $P(B/A) = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.3 & 0.7 \end{bmatrix}$.

5B. In a single input two output system, every input symbol x_i is observed as two symbols y_j & z_k respectively. It is seen that H(Y,Z/X) = 0.33 bits/symbol & H(Y,Z)=0.725 bits/symbol, H(X)=0.333bits/symbol, and mutual information of input and anyone output observing the other output are equal to 0.159 bits per symbol. Compute I(X;Y, Z) and I(X;Y;Z).

5C. Compute the probability of error of the channel described by its channel matrix shown below, using maximum likelihood decision rule. $\begin{bmatrix} 0.55 & 0.3 & 0.15 \\ 0.2 & 0.3 & 0.5 \\ 0.25 & 0.3 & 0.45 \end{bmatrix}$

(5+3+2)

6A. A transmission system is described by three binary alphabet X, Y & Z. Every x_i and y_j symbols selected at random equally likely. Every symbol in Z is defined as $z_k = x_i \oplus y_j$. Determine I(X;Y;Z) and I(X,Y,Z).

6B. A two-bit message is transmitted as five bits through a Binary Symmetric Channel with error probability 1/3 using Hamming code. Find the code words. State the decision rule. For the messages to be equally likely what is the probability of error of the decoder?

6C. Determine the non-systematic Hamming code for the message stream (0101010), Find the coding rate.

(5+3+2)

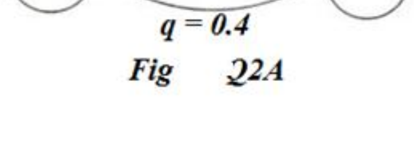

Fig Q2A

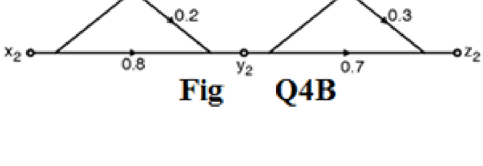

Fig Q4B