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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

A Constituent Institution of Manipal University

SEVENTH SEMESTER B.TECH. (INSTRUMENTATION & CONTROL ENGG.)
END SEMESTER EXAMINATIONS, DEC 2016/JAN 2017

SUBJECT: ROBUST CONTROL [ICE 439]

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ANY FIVE FULL** questions.
- Missing data may be suitably assumed.

1A. What do you mean by **2**-norm and ∞-norm of the system? Find the 2-norm of the system with transfer function $\mathbf{G}(\mathbf{s}) = \frac{1}{(s^3+s^2+5s+2)}$. **(5)**

1B. Find $\|\ \ \|_\infty$ and $\|\ \ \|_2$ norms of $\mathbf{G}(\mathbf{s}) = \frac{1}{s+\alpha} - \frac{2}{s+2\alpha}$ as a function of real parameter **'α'**. **(3)**

1C. Is it possible to find the norm of the system $G(S) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{10}{s+1} \end{bmatrix}$? Justify your answer. **(2)**

2A. Prove that feedback system shown in Fig. **Q2A** is internally stable if and only if the two conditions: a) the transfer function $\mathbf{1} + \boldsymbol{PCF}$ has no zeros in $\boldsymbol{Res} \geq \mathbf{0}$ and, b) there is no pole-zero cancellation in $\boldsymbol{Res} \geq \mathbf{0}$ when product $\boldsymbol{PCF}$ is formed. **(5)**

2B. Find $\frac{e}{r}$, $\frac{y}{d}$ and $\frac{y}{r}$ ratios and illustrate the sensitivity transfer function $S(s)$ and complementary sensitivity transfer function $T(s)$ for the unity feedback system as shown in Fig. **Q2A**. Also, show that $\bar{\boldsymbol{\sigma}}(\boldsymbol{S}(\boldsymbol{j\omega})) + \bar{\boldsymbol{\sigma}}(\boldsymbol{T}(\boldsymbol{j\omega})) \geq \mathbf{1}, \forall \boldsymbol{\omega}$. **(3)**

2C. Show that set of all controllers of plant $\boldsymbol{P} \in \boldsymbol{R\mathcal{H}_\infty}$ for which the feedback system shown in Fig. **Q2A** is internally stable is given by $\boldsymbol{C} = \left\{\frac{Q}{1-PQ} : \boldsymbol{Q} \in \boldsymbol{R\mathcal{H}_\infty}\right\}$ **(2)**

3A. Derive robust performance condition for the feedback control system having additive uncertainty plant model. **(5)**

3B. Assume that unity feedback system as shown in Fig. **Q2A** is internally stable and $n = \boldsymbol{d} = \mathbf{0}$. Show that if input $\boldsymbol{r}(\boldsymbol{t})$ is the unity step than $\boldsymbol{e}(\boldsymbol{t}) \to \mathbf{0}$ as $\boldsymbol{t} \to \infty$ if and only if sensitivity transfer function $\boldsymbol{S}(\boldsymbol{s})$ has at least one zero at the origin. **(3)**

3C. Consider the standard feedback loop shown in Fig. **Q2A,** where $P(s) = \frac{1}{s^2-1}$ $C(s) = \frac{s-1}{s+1}$ and $F(s) = 1,\ n = d = 0$. Is the feedback loop stable? **(2)**

4A. For the given weighting functions $\boldsymbol{W}_\Delta$ and $\boldsymbol{W}_{\boldsymbol{P}}$, derive the conditions on loop function $\boldsymbol{L}$ for the regions $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ as shown in the **Fig. Q4A** to satisfy robust performance inequality $\left\| |\boldsymbol{W}_p\boldsymbol{S}| + |\boldsymbol{W}_\Delta\boldsymbol{T}| \right\|_\infty < 1$. **(5)**

4B. Derive robust stability condition for the feedback system shown in Fig. **Q4B.** **(3)**

4C. State small gain theorem with neat diagram. **(2)**

5A. Find an internally stabilizing controller C for the plant model $P(s) = \frac{1}{(s-1)(s-2)}$ of the unity feedback system as shown in Fig. **Q2(A)** so that a) the feedback system is internally stable, b) the final value of y equals 1 when r is a unit step for $n = d = 0$ and, c) the final value of y equals zero when d is a sinusoidal signal of 10 rad/s for $n = r = 0$. **(5)**

5B. Suppose that $P(s) = \frac{1}{s}$ and $C(s) = \frac{Q}{1-PQ}$, $n = d = 0$ where Q is a proper real-rational function. Characterize those functions Q for which the feedback system shown in Fig. **Q2A** is internally stable. **(3)**

5C. Define all-pass and minimum phase transfer function with examples **(2)**

6A. Consider the unity feedback system with $C(s) = 10$ and plant $G(s) = \frac{1}{s-\alpha}$, where 'α' is real. Find the range of α for the feedback system to be internally stable. Set the nominal plant to $G_0(s) = \frac{1}{s}$ and consider α as a perturbation. Show that $G(s)$ belongs to the set of plants: $\Omega = \left\{ G = \frac{G_0(s)}{1+\Delta W G_0(s)},\ \|\Delta\|_\infty < 1 \right\}$ with $W = -\alpha$. Also, derive a condition on the closed loop system that guarantees the robust stability of the set Ω. How does this condition constrain α? **(5)**

6B Compute an internally stabilizing controller for the given plant model $\boldsymbol{G(s)} = \frac{1}{(s+1)(s+2)}$ in the unity feedback system so that output asymptotically tracks a ramp input. **(3)**

6C Consider nominal plant $\boldsymbol{P(s)} = \frac{1}{s+a_0}$ and actual plant model $\tilde{\boldsymbol{P}}(\boldsymbol{s}) = \frac{1}{s+a}$ **with** $\boldsymbol{a = a_0 + \delta\delta_m}$, $|\boldsymbol{\delta}| \leq \boldsymbol{1}$. Assume suitable weighting function $\boldsymbol{W}$ and rewrite uncertain system in the form $\tilde{\boldsymbol{P}} = \boldsymbol{P(1 + \Delta W)^{-1}}$, where Δ is stable transfer function given by $\Delta = \boldsymbol{\delta}$ with $|\boldsymbol{\delta}| \leq \boldsymbol{1}$. **(2)**

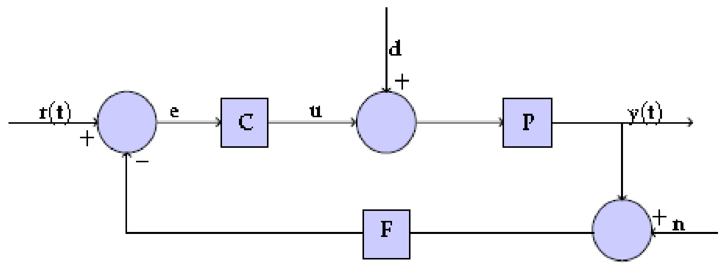

Fig. Q2A

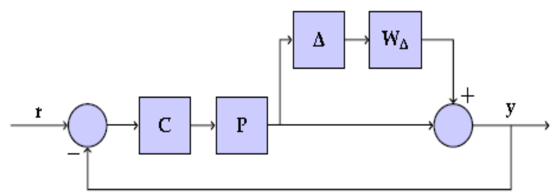

Fig. Q4B

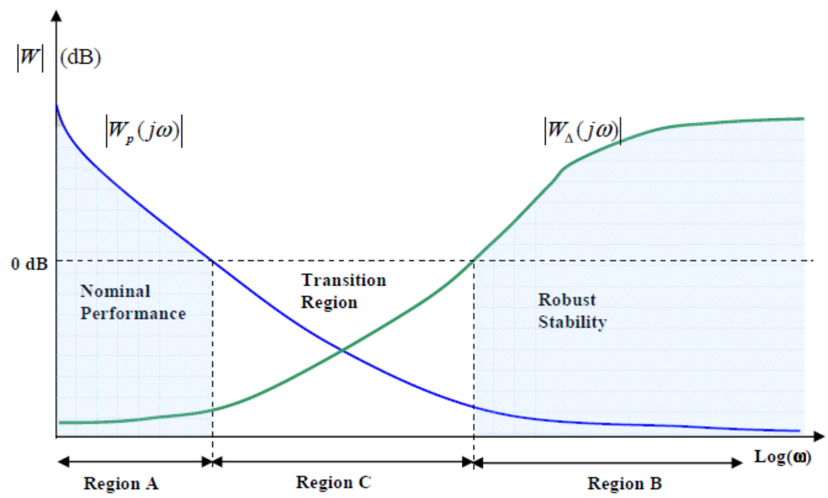

Fig. Q4A