Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

SEVENTH SEMESTER B.TECH. (INSTRUMENTATION AND CONTROL ENGG.)

END SEMESTER EXAMINATIONS, NOV/DEC 2016

SUBJECT: ROBUST CONTROL [ICE 439]

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ANY FIVE FULL** questions.
- Missing data may be suitably assumed.

1A. A plant has state space description $\dot{x} = \begin{bmatrix} -0.2 & -1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,\ y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$. Find the corresponding transfer function $G(s)$ and obtain $\| \quad \|_\infty$ of $G(s)$. Also, suggest a signal $u(t)$ for which $\|y\|_2 = \|G\|_\infty \|u\|_2$. **5**

1B. A system with transfer function $G(s) = \frac{1}{10s+1}$ has a disturbance input $d(t)$ known to have energy bound $\|d\|_2 \leq 0.4$. Find the best estimate of the ∞- norm of the output $y(t)$. **3**

1C. Compute both $\|\mathbf{y}\|_2$ and $\|\mathbf{y}\|_\infty$ for a signal $\mathbf{y}(\mathbf{t}) = \mathbf{e}^{-\mathbf{t}}$, if $\mathbf{y}$ is defined for the values of time $\mathbf{t} \in [0\ \infty)$. **2**

2A. Consider the standard feedback loop shown in Fig. **Q2A** where $P(s) = \frac{1}{s+1}$, $C(s) = K$ and $F(s) = 1$. Find the controller gain values $K > 0$ such that the steady-state absolute error $|e(t)|$ is less than or equal to 0.01 when inputs $r(t)$ are unit step and $\sin \omega t$, $0 \leq \omega \leq 4$ respectively with $n = d = 0$. **5**

2B. Assume that unity feedback system as shown in Fig. **Q2A** is internally stable and $n = d = 0$. Show that if input $r(t)$ is a ramp then $e(t) \to 0$ as $t \to \infty$ if and only if sensitivity transfer function $S(s)$ has at least two zero at the origin. **3**

2C. Consider two plants $P_1(s) = \frac{1}{s-1}$, $P_2(s) = \frac{1}{10s-1}$. Do you think one is harder to stabilize than the other? Illustrate your answer. **2**

3A. Derive robust performance condition for the feedback control system having multiplicative uncertainty plant model. **5**

3B. Consider the system $G(s) = \begin{bmatrix} \frac{1}{s+c} & \frac{4}{s+8} \end{bmatrix}$, $c \in R$ and the static controller $K = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$. What conclusion about the nominal stability of the closed loop system be deduced with the small gain theorem using ∞- norm of $G(s)$ for $c = \{-1, 1, 3\}$? **3**

3C. If $W_P(s)$ is the weighting function and $L(s)$ is the loop transfer function for the unity feedback control system, show that $\|W_P S\|_\infty < 1 \Leftrightarrow |W_P(j\omega)| < |1 + L(j\omega)|, \forall\ \omega.$ **2**

4A. Prove that the system with transfer function $P(s) = \frac{1}{s+0.5+e^{-\tau s}}$ is stable for any $\tau \in [0, 1)$. **5**

4B. A plant has transfer function $G(s) = e^{-\tau s}\frac{1}{s^2+0.4s+1}$. The delay τ is unknown, but lies between **0** and **4.** A proportional feedback controller is implemented with gain K. Use small gain theorem to determine the maximum gain K that preserves the stability for all possible values of τ and interpret your answer in terms of the Nyquist criterion. **3**

4C. Illustrate parametric uncertainty in plant model with example. **2**

5A. Find an internally stabilizing controller C for the plant model $P(s) = \frac{3}{s-4}$ of unity feedback system as shown in Fig. **Q2A** so that tracking error e goes to **0**, when r is a ramp and $n = d = 0$. **5**

5B. Derive robust stability condition for the feedback system shown in Fig. **Q5B.** **3**

5C. Show that $\left\| |W_P S| + |W_\Delta T| \right\|_\infty < 1 \Rightarrow \max_\omega \bar{\sigma}\begin{bmatrix} W_P S \\ W_\Delta T \end{bmatrix} < \frac{1}{2}$ **2**

6A. Consider $P(s) = \frac{1}{s}$ and $W_p(s) = \frac{100}{s+1}$. Perturb P to $P(s) = 1/(s+\epsilon), \epsilon > 0$. Find the controller **C** (internally stabilizing) so that $\|W_P S\|_\infty < 1$. Does the resulting controller **C** solve the performance design problem for the original **P**? Again, factor **P** as $P = P_1 P_2$, $P_1 = \frac{1}{s+1}$ $P_2 = \frac{s+1}{s}$. Solve the performance design problem for P_1; let C_1 be the solution. Set $C = C_1/P_2$. Does the resulting **C** solve the performance design problem for the original **P**? If so, explain why. **5**

6B Show that set of all controllers for which the feedback system shown in Fig. **Q2A** is internally stable is given by $C = \left\{\frac{X+MQ}{Y-NQ} : Q \in \mathcal{RH}_\infty\right\}$ **3**

6C A plant with transfer function $G = \frac{1}{0.11s^2+1.2s+1}$ is to be modelled with the nominal transfer function $G_n = \frac{1}{s+1}$. Suggest a suitable first order weighting function **W(s)** such that $G(s) = G_n(s) + \Delta W(s)$ for some Δ satisfying $\|\Delta\|_\infty \le 1$ **2**

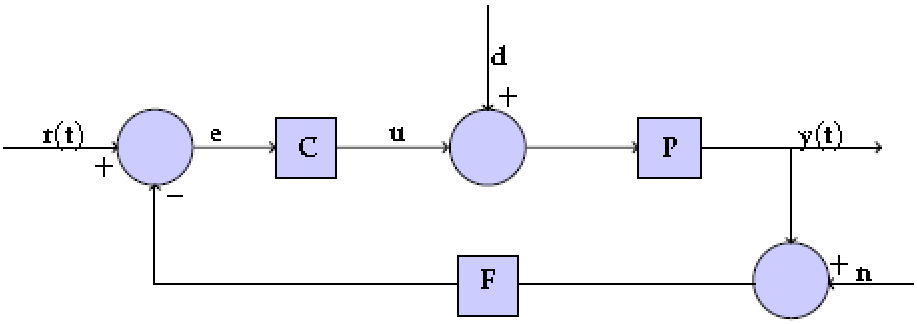

Fig. Q2A

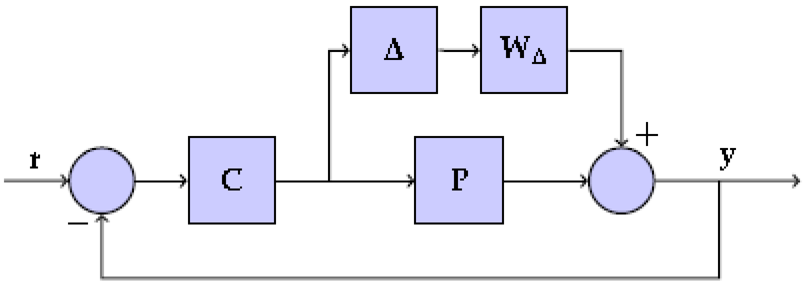

Fig. Q5B