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# Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)

## VII SEMESTER B.TECH (MECHANICAL ENGINEERING)

## END SEMESTER EXAMINATIONS, MAY- 2016

**SUBJECT: MATHEMATICAL MODELLING ON MECHANICAL SYSTEMS [MME - 445]**

**REVISED CREDIT SYSTEM**

Time: 3 Hours MAX. MARKS: 50

**Instructions to Candidates:**

- Answer **ANY FIVE FULL** the questions.
- Missing data may be suitable assumed.

1A Assume that the rate at which radioactive nuclei decay is propositional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei has undergone disintegration in a period of 100 years a) What percentage of radioactive nuclei will remain after 1000 years b) In how many years will only one fourth of the original number remain?

1B With suitable assumptions develop SIS model for epidemic and obtain the function for S(t)

1C A bacterial culture contains two strains A and B, of bacteria, with respect to population of 10 million and 16 million initially. Each strain secretes a chemical that is toxic to the other, so that in an hour, each 3 bacteria of strain A kill one bacterium of strain B and each 6 bacteria of strain B kill one bacterium of strain A. Formulate a mathematical model using difference equations. Which strain will servile and how long will it take for the other to get wiped out?

**( 3+ 3+ 4)**

2A Solve the following pay off matrix by using dominance rule. Determine the optimal strategies and value of the game.

$$\begin{array}{c} \\ PLA \end{array} \begin{array}{c} PLB \\ \begin{pmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{pmatrix} \end{array}$$

2B A salesman's territory consists of 3 cities A, B and C. He never sells in the same city of two consecutive days. If he sells in a city A, then the next day he sells in city B. However if he sells in either B or C, then the next day he is twice as likely to sell in city A as in the other city. In the long run how often does he sell in each of the cities?

2C For the two armies X and Y, the X army is about to attack Y army which has only 5000 troops while the X army has 10000 troops. The Y army however superior military equipment's which make each Y soldier 1.5 times as effective as X soldier. Develop a mathematical model by using differential equation and explain for i) which army will win ii) To estimate how many troops of the winning army will be left at the end.

**(3 + 3 + 4)**

3A A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents it is necessary to buy two products say A and B in addition. The contents of various products, per unit, in nutrient constituents are given in the following table

| Nutrients | Nutrient content in product | | Minimum amount of Nutrient required |
|---|---|---|---|
| | A | B | |
| $M_1$ | 36 | 6 | 108 |
| $M_2$ | 3 | 12 | 36 |
| $M_3$ | 20 | 10 | 100 |

If product A costs Rs 20 and B Rs 40 per unit, how much each of these two products should be bought so that total cost is minimised. Solve graphically.

3B Use simplex method to minimize P = x – 3y + 2z, subject to the constraints $3x-y+2z \le 7,\ -2x+4y \le 12,\ -4x+3y+8z \le 10,\ x \ge 0,\ y \ge 0,\ z \ge 0$

**(4 + 6)**

4A A sky driver equipped with parachute and other essential equipment falls from rest toward the earth. The total weight of the man plus the equipment is 160 lb. Before the parachute opens, the air resistance is numerically equal to $\frac{1}{2}v$, where 'v' be the velocity. The parachute opens 5 sec after the fall begins , after it opens, the air resistance is numerically equal to $\frac{5}{8}v^2$ Find the velocity i) before the parachute opens ii) after the parachute opens.

4B A police report provide the following facts: Police arrived at the scene of murder at 8 am. They immediately took and recorded the temperature of the corpse, which was 33° C and thoroughly inspected the area. By the time they finished inspection, it was 10 am. They again took the temperature of the corps, which has dropped to 29° C, and had the corpse sent to the morgue. The temperature at the crime scene had remained steady at 23° C. What is the time of death by assuming a normal body temperature of 37° C at this time? Model the situation using differential equations

4C A Lake contains 20000 fish at present. If there was no fishing the population of fish would increases by 20% every year. It is proposed to allow fishing at rate of 500 fish per year. Suppose the lake contains 100000 $m^3$ of water with 4% pollution by volume. Every day 1000 $m^3$of clean water flows into the lake and 1000 $m^3$of polluted water flow out. How long it will take for the pollution in the lake to drop to a safe level of 2% and how long does it take the pollution of fish decreases to zero.

**(3 + 3 + 4)**

5A Explain the logistic law of population growth model.

5B Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number 'x' of infected students but also to the number of students not infected, determine the number of infected students after 6 days it is further observed that after 4 days x(4) = 50

5C Explain prey predator model and discuss the equilibrium values for this model.

**(3 + 3 + 4)**

6A Solve the Game by using linear programming. The payoff matrix for player A is given by

$$\begin{array}{c} \\ PLA \end{array} \begin{array}{c} \begin{array}{ccc} PLB & & \end{array} \\ \begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{pmatrix} \end{array}$$

6B Determine the impact speed (the speed just before hitting the ground) of a bulky body falling a distance of 10 meter through air under the force of gravitation. The laws required are i) Newton's second law: The force acting upon a body is equal to the product of its mass and acceleration. ii) The force of air resistance acting on a body falling through air is propositional to the square of its velocity (with constant proportionality K = 0.218 kg/m). Assume that the acceleration due to gravity g =10 $m/s^2$

**(5 + 5)**

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