VII SEMESTER B.TECH. (MECHANICAL ENGINEERING)

END SEMESTER MAKE UP EXAMINATIONS, DEC-2016

SUBJECT: **MATHEMATICAL MODELLING ON MECHANICAL SYSTEMS [MME - 445]**

Date of Exam: 30-12-2016

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

1. Answer **any FIVE full** questions. 2. All questions carries equal marks

1A	Develop the Mathematical model for population growth. The annual birth and death rates in a country are 15.7% and 12.78% respectively, while the annual immigration and migration rates are 22.23%, 25.6%. Assuming the rates to be constant over a period of five years, use difference equation to formulate a model for population change and predict the populations of the next five years, if the current population is 685686.	**3**
1B	Two boys and two girls are throwing ball from one to other. Each boy throws the ball to the other boy with probability ½ and each girl with probability ¼. On the other hand each girl throws the ball to each boy with probability ½ and never the other girl. Find the transition probability matrix	**3**
1C	A farmer has a 100 acre farm. He can sell all the tomatoes, potatoes and beans he can raise the price to obtain Rs 1 per kg for tomatoes, Rs 0.75 per kg of potatoes and Rs 2 per kg for beans. The average yield per acre is 2000 kg of tomatoes, 3000 kg of potatoes and 1000 kg of beans. Fertilizers are available at Rs 0.50 per kg and amount required per acre is 100 kg to each of tomatoes and potatoes and about 50 kg for beans. Labour required for cultivating and harvesting per acre is 5 man day for tomatoes and beans also 6 man days for potatoes.. A total of 400 man-day of labour are available at Rs 20 per man-day. Formulate this problem as Linear programming model and maximize the former total profit by using graphical method.	**4**
2A	Eliminate the dominated strategies and solve the resulting 2x2 matrix game to find the optimal mixed strategies for the players and the value of the game. $\begin{pmatrix} 5 & -1 & 5 \\ 4 & -1 & 3 \\ -2 & 6 & 8 \end{pmatrix}$	**3**
2B	Define the equilibrium value of the dynamical system $a_{n+1} = f(a_n)$. Find the solution of the difference equation $y_{n+2} - 2\cos\alpha\, y_{n+1} + y_n = 1000$	**3**
2C	A certain chemical is converted into another chemical by a chemical reaction. The rate at which the first chemical is converted is proportional to the amount of this chemical present at any instant. Ten percent of the original amount of the first chemical has been converted in 5 min. What percent of the first chemical will have been converted in 20 min? In how minutes will 60% of the first chemical have been converted?	**4**
3A	Develop a Mathematical model for epidemic in which there are no removal and 'n' be the initial number of susceptible and 5 people has infected. Obtain the function for infected persons at time t and what happens as t tends to infinity.	**3**

3B	With suitable assumptions explain the Harrod model in economics and finance	**3**
3C	A Lake contains 20000 fish at present. If there was no fishing the population of fish would increases by 20% every year. It is proposed to allow fishing at rate of 5000 fish per year. Suppose the lake contains 100000 m^3 of water with 4% pollution by volume. Every day 1000 $m^3$of clean water flows into the lake and 1000 $m^3$of polluted water flow out. How long it will take for the pollution in the lake to drop to a safe level of 2% and how long does it take the pollution of fish decreases to zero.	**4**
4A	Define periodic state of markow chain. In a town there are two pizzerias Mario's and Luigi's. Currently, Mario's gets 42000 customers per month and Luigi's gets 63000 per month. Each month, both the dealers introduce new pizzas and offers so that 10% of Mario's customers leave and start going to Luigi's and 15% of Luigi's customers leave and start going to Mario's. Assuming that there are no new customers added to the total number. How many customers will each have after six months?	**4**
4B	A student was seen to enter a tutor's office at 4.00 pm. The tutor tutor was a later at the bar at 4.30 pm drinking heavily. At 6 pm, the cleaners discovered the student's body in the tutor's office and called the police. The police first measured the temperature of the corpse at 6.30 pm as 30^o C and later at 8.30 pm as 27^o C. The temperature of the office remained at a constant 25^oC. What is the time of death by assuming a normal body temperature of 37^o C and hence determine 'λ' the constant of proportionality	**6**
5A	A bacterial culture contains two strains A and B, of bacteria, with respect to population of 10 million and 16 million initially. Each strain secretes a chemical that is toxic to the other, so that in an hour, each 3 bacteria of strain A kill one bacterium of strain B and each 6 bacteria of strain B kill one bacterium of strain A. Formulate a mathematical model using differential equations. Which strain will servile and how long will it take for the other to get wiped out?	**4**
5B	Solve the Game by using linear programming. The payoff matrix for player A is given by $\begin{matrix} & PLB \\ PLA & \end{matrix}\begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{pmatrix}$	**6**
6A	Suppose a student carrying a flu virus returns to an isolated college campus of 2000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number 'x' of infected students but also to the number of students not infected, determine the number of infected students after 6 days it is further observed that after 4 days x(4) = 100	**4**
6B	Three grades of coal A, B and C contain phosphorous and ash as impurities. In a particular industrial process, fuel up to 100 ton (maximum) is required which should contain ash not more than 3% and phosphorus not more than 0.03% . It is desired to maximize the profit while satisfying these conditions, there is an unlimited supply of each grade. The percentage of impurities and profit of each grade are given. Find the proportion in which the three grades be used.	**6**

Coal	Phosphorus (%)	Ash (%)	Profits in Rs/Ton
A	0.02	3.0	12.00
B	0.04	2.0	15.00
C	0.03	5.0	14.00

Formulate the linear programming problem. Solve by using simplex method