Reg.No.									

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

I SEMESTER B.S. DEGREE EXAMINATION – APRIL / MAY 2017

SUBJECT: MATHEMATICS - I (MA 111)

(COMMON TO ALL BRANCH)

Friday, 5 May 2017

Time: 3 Hours **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A. If $x = \tan(log y)$, then prove that

$$(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0.$$

1B. Find a and b such that $\lim\limits_{x\to 0} \frac{x(1-a\cos x)+b\sin x}{x^3} = \frac{1}{3}$.

1C. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x dx$; n > 0. **(7 + 7 + 6)**

2A. A line makes angles $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

2B. Show that the curves $r = \frac{a}{1+cos\theta}$ and $r = \frac{b}{1-cos\theta}$ a, b>0 intersects orthogonally.

2C. Evaluate $\int_0^{2a} x^3(2ax - x^2)^{\frac{3}{2}}\, dx$. **(7 + 7 + 6)**

3A. Find the nature of the series $\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \ldots$

3B. Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$; a > 0 with explanation.

3C. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$ as a great circle. **(7 + 7 + 6)**

4A. Trace the polar curve $r = a\cos 2\theta, a > 0$ with explanation.

4B. Find the equation of the planes bisecting the angle between the planes 3x - 4y + 5z - 3 = 0 and 5x + 3y - 4z - 9 = 0. Also, specify the one which bisects the acute angle.

4C. Find the coordinates of the center of curvature at any point of the parabola $y^2 = 4ax$. **(7 + 7 + 6)**

5A. Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$.

5B. Find the area of the region to common to the regions enclosed by $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$, a>0.

5C. Prove that $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ with usual notations. **(7 + 7 + 6)**

6A. Find the entire length of the curve $x = a\cos^3\theta, y = a\sin^3\theta$, a>0.

6B. Discuss the nature of the following series.

(i) $\frac{1}{6} - \frac{1}{13} + \frac{1}{20} - \frac{1}{27} + \ . \ . \ .$ **(ii)** $\sum_{n=1}^{\infty} \frac{n}{2n-1} (-1)^n$

6C. Expand e^{sinx} in powers of x up to the terms containing x^5. **(7 + 7 + 6)**

7A. Find the image of the point (2, -3, 4) with respect to the plane 4x + 2y - 4z + 3 = 0.

7B. If $y = \frac{1}{x^2+a^2}$, then prove that $y_n = \frac{(-1)^n n!}{a^{n+2}} sin^{n+1}\theta \sin(n+1)\,\theta$.

7C. Verify Cauchy's Mean Value Theorem for f(x) = $x^3 - 3x^2 + 2$x and g(x) = $x^3 - 5x^2 + 6$x in (0, 0.5). **(7 + 7 + 6)**

8A. Find the length of the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Also find the equation of the line of shortest distance.

8B. State Lagranges's theorem and verify Rolle's Theorem for $f(x) = x(x+3)e^{-o.5x}$ in $[-3, 0]$.

8C. Find the equation of the right circular cone whose vertex is the origin, axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi vertical angle of 30°.

(7 + 7 + 6)