Reg.No.									

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

SECOND SEMESTER B.S. DEGREE EXAMINATION – APRIL/ MAY 2017

SUBJECT: MATHEMATICS II (MA 121)

(COMMON TO ALL BRANCHES)

Thursday 20 April 2017

Time: 3 Hours **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A. Find the maximum and minimum values of the function

$f(x,y) = x^3 + 3x\,y^2 - 15x^2 - 15y^2 + 72x$.

1B. Change the order of integration and hence evaluate, $\int_0^a \int_{\frac{y^2}{a}}^{2a-y} xy\,dx\,dy$.

1C. Define linearly independent and linearly dependent set of vectors. Test whether the set $B = \{(3,\ 0,\ 2),\ (7,\ 0,\ 9),\ (4,\ 1,\ 2)\}$ forms a basis for R^3. If so represent (1, 2, 3) in terms of basis vectors. **(7 + 7 + 6)**

2A. Using Gram Schmidt process construct an orthonormal set of basis vectors of $V_3(R)$ for given vectors $\{(1,\ -1,\ 0),\ (2,\ -1,\ -2),\ (1,\ -1,\ -2)\}$.

2B. Evaluate $\iint_S \left(curl\ \vec{F}\right).\ \vec{n}\ dS$, where $\vec{F} = y^2\,\vec{i} - y\,\vec{j} + xz\,\vec{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2,\ z \geq 0$.

2C. Find the area lying inside the cardioid $r = a(1 + cos\,\theta)$ and outside the circle $r = a$. **(7 + 7 + 6)**

3A. Find the angle between the tangents to the curve $x = 2t^3,\ y = 3t^2,$ $z = t$ at $t = 1$ and $t = -2$.

3B. Find the dimensions of the rectangular box open at the top of maximum capacity, whose surface area is 432 sq. cms., using Lagrange's method of undetermined multipliers.

3C. Test for consistency and solve the following equations by Gauss-elimination method. 3x + 3y + 2z = 1, x + 2y = 4, 10y + 3z = -2, 2x - 3y – z = 5. **(7 + 7 + 6)**

4A. Verify Stoke's theorem for $\vec{A} = (x^2 + y^2)\,\vec{i} - 2xy\,\vec{j}$ taken around the rectangle bounded by $x = a,\ x = -a,\ y = 0$ and $y = b$.

4B. Find the volume bounded by $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

4C. If $u = \sec^{-1}\left\{\frac{x^3y^3}{x^4+y^4}\right\}$, then prove that $xu_x + yu_y = 2cotu.$

(7 + 7 + 6)

5A. Prove that $\nabla.(u\nabla v - v\nabla u) = u\nabla^2 v - v\nabla^2 u$, where $u \; and \; v$ are scalar functions.

5B. State and Prove Greens theorem in plane .

5C. Evaluate $\iint_R x^3 y \; dx \; dy$, where R is the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.

(7 + 7 + 6)

6A. Let $u = f(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$ then, prove that $u_{xx} + u_{yy} + u_{zz} = f''(r) + \frac{2}{r} f'(r)$.

6B. Find k_1 and k_2 such that the rank of $\begin{pmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & k_1 & k_2 \end{pmatrix}$ is 3.

6C. If $(\nabla \times \overrightarrow{A}) = \vec{0}$, then evaluate $\nabla.(\vec{A} \times \vec{r})$.

(7 + 7 + 6)

7A. If the kinetic energy T is given by $T = \frac{1}{2} m v^2$. Find approximately the change in T as m changes from 49 to 49.5 and v changes from 1600 to 1590

7B. Find the volume of the region bounded by the paraboloid $x^2 + y^2 = 2z$ and the cylinder $x^2 + y^2 = 4$.

7C. Show that $\int_0^\infty \frac{x^2}{(1+x^4)^3} \; dx = \frac{5\pi\sqrt{2}}{128}$. **(7 + 7 + 6)**

8A. Using Gauss - Jordan method, find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$.

8B. Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is conservative. Find the workdone in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$.

8C. Prove that $\sqrt{\pi} \; \Gamma(2p) = 2^{2p-1} \, . \, \Gamma(p) \, . \, \Gamma\left(p + \frac{1}{2}\right)$.

(7 + 7 + 6)