Reg. No.									

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
(Manipal University)

III SEMESTER B.S. DEGREE EXAMINATION –APRIL / MAY 2017

SUBJECT: MATHEMATICS III (MA231)
(BRANCH: COMMON TO ALL)
Saturday, 06 May 2017

Time: 3 Hours **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions. Each main question carries (7+7+6) marks.**
- ✓ **Missing data, if any, may be suitably assumed.**

1A Solve $(x^3y^2 + x)dy + (x^2y^3 - y)dx = 0$.

1B Solve $y'' - 2y' + 2y = x + e^x cosx$.

1C Prove that an analytic function $f(z)$ with constant modulus is a constant.

2A Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$.

2B Find the Laplace transform of

$$F(t) = \begin{cases} \sin wt\,, & 0 < t \le \frac{\pi}{w} \\ 0\,, & \frac{\pi}{w} \le t < \frac{2\pi}{w} \end{cases} ;\ F\left(t + \frac{2\pi}{w}\right) = F(t), \forall t.$$

2C State Cauchy's integral formulae. Evaluate $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ over $|z| = 3$.

3A Find $L\left\{e^{-t}\int_0^t \frac{sint}{t} dt\right\}$.

3B Solve by method of variation of parameters $y'' - 2y' + y = e^x logx$.

3C Given the differential equation $y'' - xy' - y = 0$ with the conditions $y(0) = 1$ and $y'(0) = 0$, use Taylor series method to determine the value of $y(0.1)$.

4A Solve by the method of Laplace transforms, the equation

$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$, with $x = 2, \frac{dx}{dt} = -1$ at $t = 0$.

4B Determine the constant a such that the function $u(x, y) = \sin x \cosh ay$ is harmonic and find its harmonic conjugate.

4C Solve $y \log_e y \, dx + (x - \log_e y) dy = 0$.

5A Find the inverse Laplace transform of $\frac{s+2}{(s^2+4s+5)^2}$.

5B State residue theorem. Prove that $\int_0^\infty \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$.

5C Solve $y'' - 4y' + 4y = 8x^2 e^{2x} sin2x$.

6A Given $y' = y - x$ where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ by Runge-Kutta method of fourth order.

6B Find all possible series expansions of $f(z) = \frac{1}{(z+1)(z+3)}$ about $z = 0$.

6C Find $L\left\{\frac{cos3t - cos2t}{t} + t^2 e^{2t}\right\}$.

7A Determine the value of y when $x = 0.1$. Given that $y(0) = 1$ and $y' = x^2 + y$, with $h = 0.05$ using modified Euler's method.

7B Solve $u_{xx} + 2u_{xy} + u_{yy} = 0$ using the transformations

$v = x, \; z = x - y$.

7C Express the following function in terms of unit step function and hence find its Laplace transform: $f(t) = \begin{cases} \cos t, & 0 \le t < \pi \\ 1, & \pi \le t < 2\pi \\ \sin t, & t \ge 2\pi. \end{cases}$

8A Solve $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$.

8B Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}, a \neq b$.

8C Using the method of separation of variables, solve

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.