MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

IV Semester B.Tech. (Civil Engineering)
End Semester Examinations, April 2017
Subject: Engineering Mathematics IV [MAT 2205]
Revised Credit System

Time: 3 Hours **(24/04/2017)** MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A. Solve the partial differential equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length $h = 1$ **4M**

1B. Suppose that a random variable X has probability density function $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Find the moment generating function of X and hence find its mean. **3M**

1C. By the method of least squares, find the parabola $y = a + bx + cx^2$ that fits best for the following data: **3M**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

2A. Define the feasible solution of Linear Programming problem.
Using Simplex method, Maximize Z = $7x_1 + 5x_2$
Subject to, $2x_1 + x_2 \leq 100$, $4x_1 + 3x_3 \leq 240$,
$x_1, x_2, x_3 \geq 0$. **4M**

2B. Suppose that X is uniformly distributed over (-1, 1). Find the probability density function of $Y = 4 - X^2$. **3M**

2C. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ **3M**

3A. Explain the standard form of Linear Programming problem.
Solve by the graphical method: Maximize $z = 5x + 4y$, subject to the constraints, $6x + 4y \leq 24;\ x + 2y \leq 6;\ -x + y \leq 1;\ x, y \geq 0$. **4M**

3B.	Show that Poisson distribution as a limiting form of Binomial distribution	**3M**
3C.	In a normal distribution 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution	**3M**
4A.	Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1,\ t > 0, u(x,0) = \sin \pi x,\ u(0,\ t) = u(1,\ t) = 0$. Compute u for four time steps with $h = 0.2$.Use Schmidt's method.	**4M**
4B.	Find the curves on which the functional $\int_0^1 \left[(y')^2 + 12xy\right] dx$ with y(0) = 0 and y(1) = 1 can be extremised.	**3M**
4C.	Solve by finite difference method: $x^2 y'' + xy' + (x^2 - 3)y = 0$ with $y(1) = 0,\ y(2) = 2$, and $h = 0.25$.	**3M**
5A.	Solve by Crank-Nicolson's scheme, $\frac{\partial u}{\partial t} = \frac{1}{16} \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$ with $u(x,\ 0) = 0 = u(0,\ t),\ \ u(1,\ t) = 100t$. Compute u for one time step with $h = \frac{1}{4}$.	**4M**
5B.	State Central limit theorem. A sample of size 5 is obtained from a random variable with distribution $N(12,\ 4)$. Find the probability that the sample mean exceeds 13.	**3M**
5C.	Suppose X is binomially distributed with parameters **n** and **p**, find the mean and variance of binomial distribution	**3M**

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