Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

IV SEMESTER B.TECH. (CS/ICT/CC - ENGINEERING)

END SEMESTER MAKEUP EXAMINATIONS, JUNE-2017

SUBJECT: ENGINEERING MATHEMATICS IV [MAT 2206]

REVISED CREDIT SYSTEM

Time: 3 Hours (16/06/2017) MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A.	Each of the two persons tosses three fair coins. Find the probability that they get the same number of heads.	**3 Marks**
1B.	An archer with an accuracy of 75% fires three arrows at one target. The probability of the target falling is 0.6 if it is hit once, 0.7 if it is hit twice, and 0.8 if it is hit three times. Given that the target has fallen, find the probability that it was hit twice.	**3 Marks**
1C.	The joint pdf of the two dimensional random variable (X,Y) is given by $f(x,y) = k\ xye^{-(x^2+y^2)};\ x>0,\ y>0.$ (i). Find the value of k (ii). Prove that X and Y are independent.	**4 Marks**
2A.	Let X be a continuous random variable with pdf f given by: $f(x) = \begin{cases} ax, & 0 \le x \le 1, \\ a, & 1 \le x \le 2, \\ -ax + 3a, & 2 \le x \le 3, \\ 0, & elsewhere. \end{cases}$ (a) Determine the constant a. (b). Determine the cdf of X.	**3 Marks**
2B.	If X, Y, and Z are uncorrelated random variables with standard deviations 5, 12, and 9 respectively and if $U=X+Y$ and $V=Y+Z$, evaluate the correlation coefficient between U and V.	**3 Marks**
2C.	In a normal distribution 31% of the item are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.	**4 Marks**
3A.	Find the mean and variance of the Chi-square distribution with parameters n.	**3**

		Marks
3B.	Suppose that X is uniformly distributed over the interval (-1, 1). Obtain the pdf of the random variable $Y = \sin\left(\frac{\pi}{2}X\right)$.	**3 Marks**
3C.	If the random variable X has pdf $f(x) = \lambda e^{-\lambda(x-a)},\ x \geq a.$ Find the mgf of X and hence find $E(X)$ and $V(X)$.	**4 Marks**
4A.	State Central limit theorem. A sample of size 5 is obtained from a random variable with distribution $N(12, 4)$. Find the probability that the sample mean exceeds 13.	**3 Marks**
4B.	A random sample of size 9 from the distribution $N(\mu, \sigma^2)$ yields $s^2 = 7.63.$. Determine 95% confidence interval for σ^2.	**3 Marks**
4C.	Let $(X_1, X_2, ..., X_n)$ be a random sample of size n from a distribution having pdf $f(x;\theta) = \begin{cases} \theta^x(1-\theta)^{1-x},\ x = 0,1,2,...,\ 0 \leq \theta \leq 1 \\ 0,\quad otherwise \end{cases}$. Find the maximum likelihood estimator for θ.	**4 Marks**
5A.	A die was cast $n = 120$ independent times and the following data resulted: <table><tr><td>Spots up</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Frequency</td><td>*b*</td><td>20</td><td>20</td><td>20</td><td>20</td><td>40-*b*</td></tr></table>If we use Chi-square test, for what value of *b* would the hypothesis that the die is unbiased to be rejected at 0.025 significance level?	**3 Marks**
5B.	Let $(X_1, X_2, ..., X_n)$ denote a random sample from the distribution, that has pdf $f(x;\theta) = \begin{cases} \theta x^{\theta-1}, 0 < x < 1 \\ 0,\ otherwise \end{cases}$. Find a best critical region for testing simple hypothesis $H_0 : \theta = 1$ against alternative hypothesis $H_1 : \theta = 2.$	**3 Marks**
5C.	Let X be the random variable, having the pdf $f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta},\ 0 < x < \infty \\ 0,\ otherwise \end{cases}$. Let $H_0 : \theta = 2, H_1 : \theta = 4.$ Use random sample of size 2 and define critical region to be $C = \{(X_1, X_2) / 9.5 \leq X_1 + X_2 < \infty\}.$ Find the significance level of the test.	**4 Marks**