Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

IV SEMESTER B.TECH. (CS/ICT/CC - ENGINEERING)

END SEMESTER EXAMINATIONS, APRIL/MAY 2017

SUBJECT: ENGINEERING MATHEMATICS IV [MAT 2206]

REVISED CREDIT SYSTEM

Time: 3 Hours (24/04/2017) MAX. MARKS: 50

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A.	Consider a family of n children. Let A be the event that a family has children of both sexes, and let B be the event that there is at most one girl in the family. Find the only value of n for which the events A and B are independent (assuming that each child has a probability $\frac{1}{2}$ of being a girl).	**3 Marks**
1B.	A bag contains 3 coins out of which one coin with 2 heads and other 2 coins are normal and unbiased. A coin is chosen at random from bag and tossed 4 times in succession. If head turns out each time, what is the probability that the chosen coin is 2 headed?	**3 Marks**
1C.	Suppose that the joint pdf of the two dimensional random variable (X,Y) is given by $f(x,y)=\begin{cases} x^2+\frac{xy}{3}, & 0<x<1,\ 0<y<2 \\ 0, & elsewhere \end{cases}$. Compute, (i) $P(X+Y\geq 1)$ (ii) $E(X)$.	**4 Marks**
2A.	If the random variable K is uniformly distributed over $(0,5)$, what is the probability that the roots of the equation $4x^2+4xK+K+2=0$ are real?	**3 Marks**

2B.	Two independent random variables X_1 and X_2 have means 5 and 10, and variances 4 and 9 respectively. Find the covariance between $U = 3X_1 + 4X_2$, $V = 3X_1 - X_2$.	**3 Marks**
2C.	The annual rainfall at a certain locality is known to be a normally distributed random variable with mean equal to 29.5 inches and standard deviation 2.5 inches. How many inches of rain (annually) exceeded about 5% of the time?	**4 Marks**
3A.	Find the mean and variance of the binomial distribution with parameters n and p.	**3 Marks**
3B.	A random variable X has Cauchy's distribution with pdf $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$. Show that $Y = \frac{1}{X}$ also has Cauchy's distribution.	**3 Marks**
3C.	Let $(X_1, X_2, ..., X_n)$ be a random sample of size n from $N(\theta_1, \theta_2)$, $-\infty < \theta_1 < \infty$ and $\theta_2 > 0$. Find the maximum likelihood estimators for θ_1 and θ_2.	**4 Marks**
4A.	If $X \sim N(\mu, \sigma^2)$, then show that $E\left[(X-\mu)^{2n}\right] = 1.3.5....(2n-1)\sigma^{2n}$.	**3 Marks**
4B.	A random sample of size 17 from the distribution $N(\mu, \sigma^2)$ yields $\overline{X} = 4.7$ and $s^2 = 5.76$. Determine 90% confidence interval for μ.	**3 Marks**
4C.	A computer in adding numbers, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over $(-0.5, 0.5)$. If 1500 numbers are added, then what is the probability that the magnitude of the total error exceeds 15? How many numbers may be added together in order that the magnitude of the total error is less than 10 with probability 0.9?	**4 Marks**
5A.	Mendelian theory states that the probabilities of classification a, b, c, d are respectively $\frac{9}{16}, \frac{3}{16}, \frac{3}{16}, \frac{1}{16}$. From a sample of 160 the actual numbers observer were 86, 35, 26 and 13. Is this data consistent with the theory at 1% of significance level?	**3 Marks**

5B.	Let $(X_1, X_2, ..., X_n)$ denote a random sample from the distribution, that has pdf $f(x;\theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}}$, $-\infty < x < \infty$. Find a best critical region for testing simple hypothesis $H_0 : \theta = 0$ against alternative hypothesis $H_1 : \theta = 1$.	**3 Marks**
5C.	Let us assume that the life of a tire in miles, say X, is normally distributed with mean θ and standard deviation 5000. Past experience indicates that $\theta = 30,000$. The manufacturer claims that the tires made by new process have mean $\theta > 30,000$ and it is very possible that $\theta = 35,000$. Let us check his claim by testing $H_0 : \theta < 30,000$ against $H_1 : \theta > 30,000$. We shall observe n independent values of X say $X_1, X_2, ..., X_n$, and we shall reject H_0 (thus accept H_1) if and only if $\overline{X} \geq c$. Determine n and c so that the power function $k(\theta)$ of the test has the values $k(30,000) = 0.01$ and $k(35,000) = 0.98$.	**4 Marks**