Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

IV SEMESTER B.TECH MECHANICAL ENGINEERING END SEMESTER (MAKE-UP) EXAMINATION, JUNE 2017

SUBJECT: ENGINEERING MATHEMATICS-IV (MAT-2210)

(14-06-2017)

Time: 3 Hours | Max. Marks : 50

Answer all the questions. Statistical tables will be provided.

1A. Obtain the series solution of $4x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$.

1B. Prove that the limiting case of a Binomial distribution is a Poisson distribution.

1C. Let X be a random variable with probability distribution function $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and $\lambda > 0$. Find moment generation function of X and hence obtain $E(X)$ and $V(X)$.

(4+3+3)

2A. Let X and Y be two independent random variables with p.d.f.'s $f(x) = e^{-x}$ for $x > 0$ and $g(y) = 2e^{-2y}$ for $y > 0$ respectively. Find the p.d.f. of $Z = X + Y$.

2B. Fit a straight line of the form $y = a + bx$ to the following data:

x	1	2	3	4	5
y	14	13	9	5	2

2C. Prove that $P_n(x) = \dfrac{1}{n!2^n}\dfrac{d^n}{dx^n}\left(x^2 - 1\right)^n$.

(4+3+3)

3A. Let X, Y and Z be uncorrelated random variables with standard deviation 5,12 and 9 respectively. If $U = X + Y$ and $V = Y + Z$, evaluate the correlation coefficient between U and V.

3B. The life length T of an electronic device follows a p.d.f. $f(t) = 0.001e^{-0.001t}$. Suppose that 100 such devices are tested, yielding observed values $T_1, T_2, ..., T_{100}$. Find $P(950 < \overline{T} < 1100)$.

3C. Prove that $\frac{d}{dx}\left(x^{-n}J_n(x)\right) = -x^{-n}J_{n+1}(x)$, hence deduce that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

(4+3+3)

4A. In an examination taken by 600 candidates, the average and the standard deviation of marks obtained are 40% and 10%. Find (i) How many will pass if 45% is fixed as a minimum? (ii) What should be the minimum if 400 candidates are to pass?

4B.A basket contains 40 mangoes, out of which 15% are overripe. Two mangoes are selected at random. If both are good, then the basket is accepted. If one is good and the other is overripe, then another is picked from the remaining and if this is good, then the basket is accepted. Find the probability of accepting the basket.

4C. The joint pdf of 2 dimensional random variable is given by

$$f(x,y) = \begin{cases} x^3 + \frac{xy}{3}, & \text{if } 0 < x < 1, 0 < y < 2 \\ 0, & \text{else where} \end{cases}$$

Compute (i) $P(X + Y \geq 1)$ (ii) $P(Y < X)$.

(4+3+3)

5A. A binary communication channel carries data as one of two types of signals denoted by 0 and 1. Due to noise a transmitted 0 is received as 1 with probability 0.6 and 1 is received as 0 with probability 0.09. Probability of transmitting 0 is 0.45. If a signal is sent then determine the (i) probability that 1 is received (ii) probability that 0 was transmitted given that 0 was received.

5B. If X is a continuous random variable with the p.d.f. $f(x) = \frac{x}{a^2}e^{\frac{-x^2}{a^2}}$ for $x > 0$ then, find $E(X)$ and $V(X)$.

5C. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained and let Y be the number of queens obtained. (i) Obtain the joint probability distribution of (X, Y). (ii) Obtain the marginal distribution of X and Y.

(4+3+3)

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