Reg. No. | | | | | | | | | |

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

IV SEMESTER B.TECH MECHANICAL ENGINEERING END SEMESTER EXAMINATION, APRIL 2017

SUBJECT: ENGINEERING MATHEMATICS-IV (MAT-2210)

(21-04-2017)

Time: 3 Hours Max. Marks : 50

Answer all the questions. Statistical tables will be provided.

1A. A paint store chain produces and sells latex and semi gloss paint. Based on long range sales, the probability that a customer will purchase latex is 0.75. Of those that purchase latex paint, 60% also purchase rollers. But 30% of semi gloss buyers purchase rollers. A randomly selected buyer purchase a roller and a can of paint. What is the probability that the paint is latex?

1B. Fit a parabola of the form $y = a + bx + cx^2$ to the following data:

x	0	1	2	3	4	5
y	1	3	7	13	21	31

1C. A bag contains 10 white balls and 3 red balls while another bag contains 3 white balls and 5 red balls. Two balls are drawn at random from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball?

(4+3+3)

2A. Find the power series solution of the differential equation $\dfrac{d^2y}{dx^2} - 2xy = 0$.

2B. If X is a random variable taking values $0, 1, 2, ...$ and $P(X = x) = pq^x$, where p and q are positive constant such that $p + q = 1$ then, find the moment generating function (m.g.f.) of X. If $E(X) = m_1$, $E(X^2) = m_2$, show that $m_2 = m_1(2m_1 + 1)$.

2C. Suppose that the random variable X has possible values $1, 2, 3, ...$ and the probability distribution $P(X = x) = \dfrac{1}{2^j}$ for $j = 1, 2, 3, ...$ Find (i) $P(X$ is even$)$
(ii)$P(X$ is divisible by 3$)$.

(4+3+3)

3A. The marks X obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine,

(i) how many students got marks above 98%.

(ii) what was the highest marks obtained by the lowest 10% of the students.

3B. Let X be a random variable follows normal distribution with mean μ and variance σ^2. Find the p.d.f of $Y = \left(\frac{x-\mu}{\sigma}\right)^2$.

3C. Prove that $x^4 - 3x^2 + x = \frac{8}{35}P_4(x) - \frac{10}{7}P_2(x) + P_1(x) - \frac{4}{5}P_0(x)$.

(4+3+3)

4A. A two dimensional random variable (X, Y) is uniformly distributed in the region bounded by a circle $x^2 + y^2 = a^2$. Find $COV(X, Y)$.

4B. Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3-x^2}{x^2}\right)\sin x - \frac{x}{2}\cos x\right]$

4C. Derive mean and variance of a Poisson distribution.

(4+3+3)

5A. If X and Y are two independent random variables with p.d.f.'s $g(x) = 2e^{-x^2}$ for $x \geq 0$ and $h(y) = 2e^{-y^2}$ for $y \geq 0$ respectively. Then find the p.d.f. of $R = \sqrt{X^2 + Y^2}$.

5B. Let $X_1, X_2, ..., X_{25}$ and $Y_1, Y_2, ..., Y_{25}$ be two independent samples taken from $N(3, 16)$ and $N(4, 9)$ respectively. Find $P\left(\frac{\overline{X}}{\overline{Y}} > 1\right)$

5C. A box contains 12 items of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X and $V(X)$

(4+3+3)

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