Reg. No.										

DEPARTMENT OF SCIENCES, M.Sc (APPLIED MATHEMATICS & COMPUTING)
II SEMESTER END SEMESTER EXAMINATIONS, APRIL 2017

SUBJECT: COMPLEX ANALYSIS (MAT - 604)
(REVISED CREDIT SYSTEM)

Time: 3 Hours Date: 21-04-2017 MAX. MARKS: 50

Note: (i) a) Answer any FIVE full questions. b) All questions carry equal marks.

1A. State and Prove Casorati-Weierstars theorem.

1B. For any complex number a_i and b_j prove that $\left|\sum_{i=1}^{n} a_i b_i\right|^2 \leq \sum_{i=1}^{n}|a_i|^2 \sum_{i=1}^{n}|b_i|^2$

1C. (i) If S is a Mobius Transformation then show that S is a Composition of translation, dilation and the inversion

(ii) If $f(z)= \begin{cases} \dfrac{\bar{z}^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ then show that C-R equation are not sufficient for the differentiability of f. **(3 + 3+ 4)**

2A. Find the bilinear transformation which maps the points z = 1, i, - 1 onto the points w = i, 0, -i . Hence find the invariant points.

2B. Derive Cauchy-Riemann equations in Cartesian form for an analytic function f(z) = u+ i v and hence show that u and v are harmonic.

2C. . Evaluate : (i) $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$ (ii) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx$

(3 + 3+ 4)

3A. Determine the analytic function f(z) if $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$

3B. Define the residue of a function at an isolated singularity, state and prove residue theorem.

3C. (i) Prove that : If the power series $\sum_{n=0}^{\infty} a_n z^n$ converges for a particular value z_0 of z, it converges absolutely for all values of z for which $z| < |z_0|$

(ii) Prove that : if f(z) and g(z) are analytic inside and on a simple closed curve C and if $|g(z)| < |f(z)|$ on C, then f(z) + g(z) and f(z) have the same number of zeros inside C . **(3 + 3+ 4)**

4A. Show that the transformation w = sin z maps lines parallel to the coordinate axes in the Z-plane into conformal conics.

4B. Prove that : If is f(z) is analytic at z_0 then f is conformal at z_0 provided $f'(z_0) \neq 0$

4C. State and prove Liouville's theorem and hence establish the fundamental theorem of algebra.

(3 + 3+ 4)

5A. Find all Taylors and Laurent series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ about the origin

5B. (i) State Riemann Mapping theorem

(ii) Prove that the complex plane C and the unit disc are homeomorphic but not isomorphic.

5C. State and prove maximum principle and hence establish Schwarz lemma.

(3 + 3+ 4)

6A. State Cauchy –Goursat theorem for a triangle and hence evaluate :

$$\int_{\gamma} \frac{e^{2z}\, dz}{(z+1)^3(z-2)}$$ where γ is the circle $|z|=3$.

OR

6A. Prove that : if f(z) be continuous in a simply connected domain D and $\oint_{\gamma} f(z)\,dz = 0$ where γ is any rectifiable Jordan curve in D then f(z) is analytic in D.

6B. If $f : A \to C$ be analytic on an open set A of C then show that for all $\forall z \in A$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|u|^p = p(p-1)|u|^{p-2}\,|f'(z)|^2$$

6C. State and Prove Taylors theorem

(3 + 3+ 4)

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