MANIPAL UNIVERSITY

Reg. No.										

Deemed- to -be -University under Section 3 of the UGC Act, 1956

DEPARTMENT OF SCIENCES,
M. Sc. APPLIED MATHEMATICS AND COMPUTING

II SEMESTER END SEMESTER MAKEUP EXAMINATIONS, JUNE 2017

SUBJECT: TOPOLOGY AND FUNCTIONAL ANALYSIS [MAT 605]

(REVISED CREDIT SYSTEM)

Time: 3 Hours | Date: 13 -06 -2017 | MAX. MARKS: 50

Note : **(i) Answer any FIVE full questions** **(ii) All questions carry equal marks**

1A. In topological space (X, τ), A is a subset of X then prove that

(i) $\partial A \cap A = \phi$ *iff A is open* where ∂A is boundary points of A.

(ii) $\partial A \subset A$ *iff A is closed*

1B. In a topological space (X, τ) if A and B are subsets of X then show that

$$(A \cap B)^o = A^o \cap B^o$$

1C. If $A \subset Y$ where (Y, τ_y) is a subspace of (X, τ) then show that A is τ_y - closed iff A is the intersection of Y and τ – closed set. (3 + 3 + 4)

2A. For the T_1 space (X, τ), $E \subset X$. Prove that a point x in X is a limit point of E then every open set containing x contains infinitely many points of E.

2B. If A, B are subsets of a topological space (X, τ) and d(A) is the derived set of A then prove that $d(A \cup B) = d(A) \cup d(B)$.

2C. If f is function from a topological space X onto a topological space Y then following statements are equivalent :

(i) f is a homeomorphism

(ii) f is continuous, one-one, onto and an open mapping

(iii) f is continuous, one-one, onto and an closed mapping (3 + 3 + 4)

3A. In topological space (X, τ) a subfamily β of τ is a base for τ iff for each member U of τ and each point $x \in U$, there is a member B in β such that $x \in B$ and $B \subset U$.

3B. Prove that continuous image of a compact set is compact.

3C. State and prove Baire's Category theorem. (3 + 3 + 4).

4A. For any two vectors x and y in a Banach space, prove that $\left| \, \|x\| - \|y\| \, \right| \le \|x - y\|$. Hence deduce that the norm is a continuous function.

4B. For the normed linear spaces N and N′, consider the normed linear space B(N, N′) of all continuous linear transformations of N into N′ with the norm defined by $\|T\| = \sup\{\|T(x)\| : \|x\| \le 1\}$. If N′ is complete, then show that the normed linear space B(N, N′) is also complete.

4C. Let N and N′ be normed linear spaces and T be a linear transformation of N into N′. Show that there exists a real number K ≥ 0 with the property that $\|T(x)\| \le K\|x\|$, for every $x \in N$ if and only if T(S) is a bounded set in N′, where $S = \{x : \|x\| \le 1\}$ is the closed unit sphere in N.

(3 + 3 + 4).

5A. State and prove Hahn-Banach theorem.

5B. If B and B′ are Banach spaces, and if T is a continuous linear transformation of B into B′, then show that T is an open mapping.

5C. Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

(3 + 3 + 4).

6A. Show that an operator T on a complex Hilbert space H is self adjoint if and only if $\langle Tx, x \rangle$ is real for all x in H.

6B. If T* denotes the adjoint of an operator T on a Hilbert space H, show that

(i) $(T_1+T_2)^* = T_1^*+T_2^*$ (ii) $T^{**} = T$ (iii) $\|T^*\| = \|T\|$.

6C. If x and y are any two vectors in a Hilbert space, then show that $|\langle x, y \rangle| \le \|x\| \, \|y\|$.

(3 + 3 + 4).

******* ALL THE BEST *******