MANIPAL UNIVERSITY

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Reg. No.									

DEPARTMENT OF SCIENCES, M.Sc(Applied Mathematics and Computing)
IV SEMESTER END SEMESTER EXAMINATIONS, APRIL 2017

SUBJECT: STOCHASTIC PROCESSES AND RELIABILITY [CODE :MAT 706]

(REVISED CREDIT SYSTEM)

Time: 3 Hours **Date:24/04/2017** **MAX. MARKS: 50**

Note: (i) Answer All FIVE full questions

(ii) All questions carry equal marks (3 + 3+ 4)

1A. Define Generating function.Let X be a random variable denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Show that p.g.f of X is $[(s^2/4)\{1 - s/2 - (s/2)^2\}^{-1}]$.

1B. Consider a sequence $\{X_n\}$ of independent coin-tossing trials with probability p for head H in a trial. Denote the states of X_n by states 1, 2, 3, 4 according as the trial numbers (n-1) and n result in HH,HT,TH,TT respectively. Show that $\{X_n\}$ is a Markov chain . Find (i) P ($X_1 = 3$ / $X_3 = 3$) (ii) P ($X_3 = 3$ / $X_1 = 4$).

1C. If $N_1(t)$, N_2 (t) are two independent Poisson processes with parameters λ_1, λ_2 respectively , then find the distribution of $P\{N_1(t) = k \mid N_1(t) + N_2(t) = n\}$. Hence write it's mean and variance.

2A. State and derive Yule – Fury birth process.

2B. Define the followings:
(i) Immigration – Emigration Process (ii) Immigration – Death Process
(iii) Time dependent Poisson Process

3A. Find the probability of ultimate extinction in the case of the linear growth process starting with i individuals at time 0.

3B. Prove that the state j is persistent or transient according as $\sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty$ or $< \infty$

3C. Consider the Markov chain with t.p.m

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 2 & 0 & 1 & 0 \end{array}$$

Find the mean recurrence time for the state 2. Is the chain irreducible, ergodic? If so find the limiting distributions.

4A. Suppose that customers arrive at a counter with a mean rate of 2/min. Find the probability that the interval between two successive arrival is

(i) more than 2 min (ii) 5 min or less (iii) between 2 & 6 min.

4B. Consider the process

$X(t) = A \cos wt + B \sin wt$, where A ,B are uncorrelated random variables with mean 0 and variance 1 and w is a positive constant.

4C. Suppose that a fair die is tossed .Let the states of X_n be k(=1,2,……,6),where k is the maximum number shown in the first n tosses. Find P and $p(X_2=6)$.

5A. The number of accidents in a town follows a Poisson process with a mean of two per day and the number X_i of people involved in the i'th accident has the distribution $P\{X_i=k\}=1/2^k (k\geq 1)$.Find the mean and variance of the number of people involved in accidents per week.

5B. State and prove chapman – Kolomogorov's equation.

5C. Find the differential equation of pure death process. If the process starts with i individuals, find the mean and variance of the number N(t) present at time t.

6A. Find the probability of ultimate extinction in the case of the linear growth process starting with *i* individuals at time 0.

6B. Solve the difference equation

$u_n = q\, u_{n-1} + p(1 - u_{n-1})$, $n\geq 1$, $p + q = 1$ and $u_0 = 1$ using p.g.f

6C. Prove that the state j is persistent or transient according as $\sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty$ or $< \infty$

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