MANIPAL UNIVERSITY

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Reg. No.										

DEPARTMENT OF SCIENCES, M. Sc. (P/C/M/G)
II SEMESTER END SEMESTER EXAMINATIONS
JUNE 2017
Subject: Quantum Mechanics II (PHY-606)
(REVISED CREDIT SYSTEM)

Time: 3 Hours Date: June 2017 MAX. MARKS: 50

Note: (i) Answer any five full questions.
(ii) Answer the questions to the point.

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1. (i) If the eigenvalues of J^2 and J_z are given by $J^2|\lambda, m>= \lambda|\lambda, m>$ and $J_z|\lambda, m>= m|\lambda, m>$. For the simultaneous eigenvector of J^2 and J_z, $|\lambda, m>$ show $\lambda \geq m^2$. Take $\hbar = 1$. [3]
(ii) Show that $\vec{L} \times \vec{L} = i\vec{L}$. [4]
(iii) Determine the orbital momenta of two electrons in the configuration $p^1 d^1$. [3]
2. (i) Calculate the energy correction due to spin-orbit coupling for hydrogen atom. [5]
(ii) Given the matrix for H^0 and H' as $\begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix} \begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix}$ in the orthonormal basis $|1>$ and $|2>$, determine the
(a) energy eigenvalues [2]
and (b) energy eigenfunctions. [3]
3. (i) The result of the variational method always gives an upper limit for the ground state energy of the system. Why? [4]
(ii) Use the WKB method to calculate the transmission coefficient for the potential barrier

$$V(x) = \begin{cases} V_0 - ax, & x > 0; \\ 0, & x < 0. \end{cases} \qquad [6]$$

4. (i) Use time dependent perturbation theory to obtain an expression for the transition in first order approximation. [5]
(ii) Calculate the electric dipole transition moment $< 2p_z|\mu_z|2s >$ for the $2s \to 2p_z$ transition in a hydrogen atom. [5]
5. (i) Obtain the expression of differential scattering cross-section in terms of beam luminosity. [4]

(ii) In scattering from a potential V(r); the wave function $\psi(r)$ is written as an incident plane wave plus an outgoing scattered wave: $\psi = exp(ikz) + f(r)$. Derive a differential equation for $f(r)$ in the first Born approximation. [6]

6. (i) Explain how the KG equation leads to positive and negative probability density values? [5]

(ii) Obtain the plane wave solutions of the Dirac equation. [5]

Useful formulae:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

$$\int_0^\infty exp(-a^2x^2)cos(bx)\,dx = \frac{\sqrt{\pi}}{2a}exp\left(-\frac{b^2}{4a^2}\right)$$

$$\int_0^\infty x^n exp(-ax)\,dx = \frac{n!}{a^{n+1}}, \quad \textbf{where} \quad n \geq 0, \quad a > 0$$

For hydrogen atom:

$$\psi_{1s} = \left(\frac{1}{\pi a_0^3}\right)^{\frac{1}{2}} exp\left(-\frac{r}{a_0}\right)$$

$$\psi_{2s} = \left(\frac{1}{32\pi a_0^3}\right)^{\frac{1}{2}} \left(2 - \frac{r}{a_0}\right) exp\left(-\frac{r}{2a_0}\right)$$

$$\psi_{2p_z} = \left(\frac{1}{32\pi a_0^3}\right)^{\frac{1}{2}} \frac{r}{a_0} exp\left(-\frac{r}{2a_0}\right) cos\theta$$