MANIPAL UNIVERSITY

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DEPARTMENT OF SCIENCES, M. Sc. (P/C/M/G)
II SEMESTER END SEMESTER EXAMINATIONS
APRIL 2017
Subject: Quantum Mechanics II (PHY-606)
(REVISED CREDIT SYSTEM)

Time: 3 Hours   Date: 25 April 2017   MAX. MARKS: 50

Note: (i) Answer any five full questions.
(ii) Answer the questions to the point.

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1. (i) Prove that $J_+|\lambda, m>= C_+|\lambda, m+1>$, where C_+ is a constant. [4]
(ii) The Hamiltonian of a system of three nonidentical spin-half particles is

$$H = A\vec{S}_1 \cdot \vec{S}_2 - B(\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3$$

where A and B are constants and $\vec{S}_1$, $\vec{S}_2$, and $\vec{S}_3$ are the spin angular momentum operators. Find their energy levels and their degeneracies. [6]
2. (i) Calculate the second order energy correction for a non-degenerate perturbative system. [5]
(ii) A particle in a central potential has an orbital angular momentum quantum number $l = 3$. If its spin $s = 1$, find the energy levels and degeneracies associated with the spin-orbit interaction. [5]
3. (i) The result of the variational method always gives an upper limit for the ground state energy of the system. Why? [4]
(ii) Use the WKB method to calculate the transmission coefficient for the potential barrier

$$V(x) = \begin{cases} V_0 - ax, & x > 0; \\ 0, & x < 0. \end{cases} \qquad [6]$$

4. (i) Use time dependent perturbation theory to obtain an expression for the transition in first order approximation. [5]
(ii) Calculate the electric dipole transition moment $< 2p_z|\mu_z|2s >$ for the $2s \to 2p_z$ transition in a hydrogen atom. [5]
5. (i) Obtain the expression of differential scattering cross-section

in terms of beam luminosity. [4]

(ii) In scattering from a potential V(r); the wave function $\psi(r)$ is written as an incident plane wave plus an outgoing scattered wave: $\psi = exp(ikz) + f(r)$. Derive a differential equation for $f(r)$ in the first Born approximation. [6]

6. (i) Why do we need a separate quantum theory for relativistic systems? What are the different quantum approaches for relativistic quantum mechanics? [3]

(ii) Consider the one dimensional Dirac equation:

$$i\hbar\frac{\partial\psi}{\partial t} = [c\alpha p_z + \beta mc^2 + V(z)]\psi; \qquad p_z = -i\hbar\frac{\partial}{\partial z}$$

$$\alpha = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

Show that (i) $\sigma = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$ **commutes with** H **(ii) the one dimensional Dirac equation can be written as two coupled first order differential equations. [7]**

Useful formulae:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

$$\int_0^\infty exp(-a^2x^2)cos(bx)\,dx = \frac{\sqrt{\pi}}{2a}exp\left(-\frac{b^2}{4a^2}\right)$$

$$\int_0^\infty x^n exp(-ax)\,dx = \frac{n!}{a^{n+1}}, \quad \textbf{where} \quad n \geq 0, \quad a > 0$$

For hydrogen atom:

$$\psi_{2s} = \left(\frac{1}{32\pi a_0^3}\right)^{\frac{1}{2}}\left(2 - \frac{r}{a_0}\right)exp\left(-\frac{r}{2a_0}\right)$$

$$\psi_{2p_z} = \left(\frac{1}{32\pi a_0^3}\right)^{\frac{1}{2}}\frac{r}{a_0}exp\left(-\frac{r}{2a_0}\right)cos\theta$$