Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

SECOND SEMESTER M.TECH. (CONTROL SYSTEMS)

END SEMESTER MAKE UP EXAMINATIONS, JUNE 2017

SUBJECT: OPTIMAL CONTROL [ICE 5234]

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A. State and explain the requirements to formulate an optimal control problem. **3**

1B. Consider the discrete time system described by **7**

$x(k+1) = x(k) + u(k); \quad k = 0, 1$

Determine an optimal control law u(0), u(1) to minimize the performance measure

$J = x^2(2) + 2u^2(0) + 2u^2(1)$

subject to the constraints

$0.0 \le x(k) \le 1.5; \quad k = 0, 1, 2$ and $-1.0 \le u(k) \le 1.0; \quad k = 0, 1.$

2A. Find the extremals for the functional $J(x) = \int_0^{\frac{\pi}{2}} [\dot{x}_1^2(t) + \dot{x}_2^2(t) + 2x_1(t)x_2(t)]dt$. The terminal conditions are $x_1(0)=0$, $x_2(0)=0$, $x_1(\frac{\pi}{2})=1$, $x_2(\frac{\pi}{2})= -1$. **6**

2B. Define increment and variation of a functional with an example. **4**

3A. A first order linear system $\dot{x} = -10x(t) + u(t)$ is to be controlled to minimize the Performance measure $J = \frac{1}{2}x^2(0.04) + \int_0^{0.04} \left[\frac{1}{4}x^2(t) + \frac{1}{2}u^2(t)\right]dt$. The admissible state and control values are not constrained by any boundaries. Find the optimal control law using the H-J-B equation. **6**

3B. State and prove the fundamental theorem of calculus of variation. **4**

4A. Determine the optimal control law by solving reduced matrix Riccati equation for the system $\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \quad y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} X(t)$ such that the following performance index is minimized. $J = \int_0^{\infty} [y_1^2(t) + y_2^2(t) + u^2(t)]dt$. Compute the open loop and closed loop eigen values and comment on the closed loop response. **5**

4B. Derive the necessary equation to be solved to obtain optimal control law for linear state regulator problem. **5**

5A. Using Hamiltonian method, derive the expression for optimal control law which minimizes the performance measure $J = \frac{1}{2}\int_0^2 \left(x^2(t) + u^2(t)\right)dt$, for the system described by **7**

$\dot{x}(t) = -x(t) + u(t);$ with x(0)=1, x(2) free

a) when admissible controls are not bounded

b) When the admissible controls are constrained with $|u(t)| \leq 0.5$.

5B. Define minimum control effort problem. Hence derive the form of optimal control for a class of minimum fuel problems. **3**

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