Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

A Constituent Institution of Manipal University

SECOND SEMESTER M.TECH. (CONTROL SYSTEMS)

END SEMESTER EXAMINATIONS, APRIL/MAY 2017

SUBJECT: OPTIMAL CONTROL [ICE 5234]

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A. With relevant expressions classify the performance measures used in optimal control. **3**

1B. Consider the system description **7**
x(k+1)=0.75*x(k)+u(k).
It is desired to bring the system state to the target set S defined by 0.0≤x(2)≤1.0, with minimum expenditure of control effort, that is minimize
J=$u^2(0)+u^2(1)$.
The allowable state and control values are constrained by
0.0≤x(k)≤3.0,
-0.5.0≤u(k)≤0.5.
Quantize the state values into the levels x(k)=0,1,2,3 for k=0,1,2 and the control values into the levels u(k)=-0.5,0,0.5 for k=0,1.
What is the optimal control sequence {$u^*(0)$, $u^*(1)$,} , if x(0)=3.0?

2A. Derive the Hamilton-Jacobi-Bellman equation used to formulate necessary condition for the solution of optimal control problem. **5**

2B. Determine the optimal control law using Riccati equation for the system $\dot{x}(t) = -x(t) + u(t)$. The performance measure is J=$\int_0^1 \frac{1}{2}\left[3x^2(t) + u^2(t)\right]dt$. The admissible controls are not bounded. **5**

3A. A first order system is described by the differential equation $\dot{x}(t) = x(t) + u(t)$. It is desired to find the control law that minimizes the performance measure **5**
J=$\frac{1}{4}x^2(T) + \int_0^T \frac{1}{4}\left[u^2(t)\right]dt$. Use HJB approach.

3B. Find the optimal control $u^*(t)$, that minimizes the performance index J=$\int_0^1 \frac{1}{2}\left[u^2(t)\right]dt$, and drives the system $\dot{x}(t) = -x(t) + u(t)$ from the initial state x(0)=5 to the final state x(1)=0, using variational calculus method. **5**

4A. Formulate the necessary conditions along with boundary conditions for determining the **6**

extremal $x^*(t)$ for a functional J= $\int_{t_o}^{t_f} g(x(t), \dot{x}(t), t)dt$ with t_0, $x(t_0)$, t_f specified and $x(t_f)$ free.

4B. Find the extremal $x^*(t)$ that minimizes the functional J(x)= $\int_0^1 \left[\frac{1}{2}\dot{x}^2(t) + 3x(t)\dot{x}(t) + 2x^2(t) + 4x(t)\right]dt$ and passes through the points x(0)=1, x(1)=4. **4**

5A. Formulate the linear state regulator problem using Hamiltonian approach and derive the necessary equations to obtain the optimal control law. **5**

5B. State and explain Pontryagin's minimum principle. **3**

5C. Define Variation. Determine variation of the functional J(x)= $\int_{t_0}^{t_f} \left[x^3(t) - x^2(t)\dot{x}(t)\right]dt$ **2**

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