



TIME: 3 HOURS

MAX. MARKS: 50

Instructions to candidates

- Answer **ANY FIVE** full questions.
- Missing data may be suitably assumed.

1A. Solve the following set of linear equations using Gauss-Jordan method.

$$x_1 - 3x_3 = -2$$

$$3x_1 + x_2 - 2x_3 = 5$$

$$2x_1 + 2x_2 + x_3 = 4$$

1B. Consider a LTI system with impulse response $h(n) = u[n] - u[n-3]$. If the applied input is $x(n) = \cos(n\pi/2)$, compute the response of the LTI system for $N=4$ using matrix multiplication method.

1C. Compute L2-norm distance between X and Y.

$$X = [1+j3 \quad 1-j \quad 3+j2]$$

$$Y = [j \quad 2-j \quad 1+j]$$

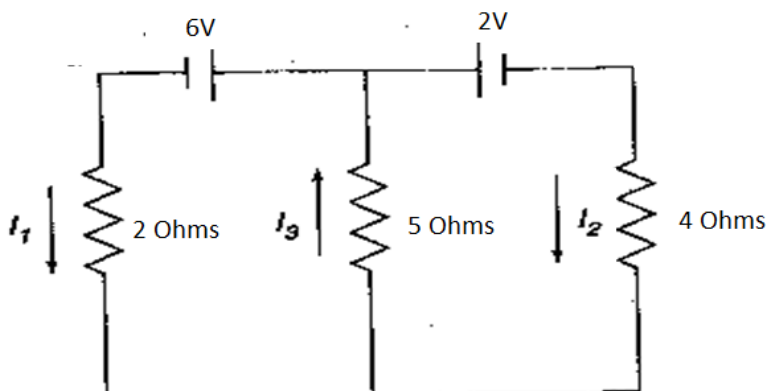
(5+3+2)

2A.

Define a linear vector space along with all its properties. Determine if $x = \begin{bmatrix} 6 \\ 7 \\ 1 \\ -4 \end{bmatrix}$ is in the subspace

spanned by the columns of A, where $A = \begin{bmatrix} 5 & -5 & -9 \\ 8 & 8 & -6 \\ -5 & -9 & 3 \\ 3 & -2 & -7 \end{bmatrix}$

2B. Find the currents in the following circuit using linear algebra.



2C. Define the following (i) Jacobian of a matrix (ii) Positive semidefinite matrix

(5+3+2)

3A. Discuss the method of estimating narrow band signal frequency buried in white noise using signal subspace decomposition. If observed signal is $x(n) = Ae^{j\omega n} + v(n)$ {where A = amplitude, ω = angular frequency, $v(n)$ is noise} and the autocorrelation matrix is estimated to be

$$\begin{bmatrix} 8 & 2-j3 \\ 2+j3 & 8 \end{bmatrix}. \text{ Estimate the signal frequency and noise variance.}$$

3B. Find the values of K for which following system of linear equations have (i) unique solution (ii) no solution (iii) infinite solutions.

$$Kx + y + z = 1$$

$$x + Ky + z = 1$$

$$x + y + Kz = 1$$

3C. Define the following (i) Hessian of a matrix (ii) Toeplitz matrix

(5+3+2)

4A. Solve the following system of linear equations using QR factorization.

$$x - 3y + z = 4$$

$$2x - 8y + 8z = -2$$

$$-6x + 3y - 15z = 9$$

4B. Using the expression for generalized Fourier Series, derive the expression for continuous Fourier series.

4C. Define Hankel matrix and mention its applications in signal processing.

(5+3+2)

5A. Find an orthonormal basis for the following subspace of \mathbb{R}^4

$$\begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

5B. Find the standard matrix for the stated composition of linear operators on \mathbb{R}^3 . A rotation of 30° about the x-axis, followed by a rotation of 30° about the z-axis, followed by a contraction with factor 0.25.

5C. Eigen values of a Hermitian matrix are _____ and Eigen vectors are _____.

(5+3+2)

6A. Discuss the use of singular value decomposition in edge detection of RGB images.

6B. Find SVD of $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

6C. Define vector space isomorphism.

(5+3+2)