



TIME: 3 HOURS

MAX. MARKS: 50

Instructions to candidates

- Answer **ALL** questions.
- Missing data may be suitably assumed.

1A. Consider a road network shown in Figure 1A. The streets are one way and the flow of traffic is measured in vehicles per hour (vph).

- Write the following traffic as system of linear equations in $Ax=b$ form.
- Find the LU decomposition of matrix A .
- What is the minimum value of x_3 that would not lead to traffic congestion?

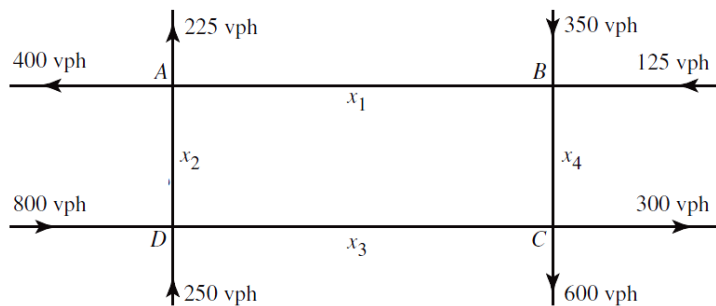


Figure 1A

1B. Consider the following basis for \mathbb{R}^2 :

$$E = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$

- Find the coordinates for the vector $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ in terms of the basis E .
- Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the following linear transformation: $T(x, y) = (2x - y, 3x - 2y)$. Find the matrix representing T with respect to the basis E .

1C.

For what value of 'x' will the matrix given becomes singular? $\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$

(5+3+2)

2A. In a certain experiment, the first five measurements of the two quantities x and y are given by $(1, 0)$, $(2, 3)$, $(3, 7)$, $(4, 14)$, $(5, 22)$. Due to random errors in the measurements the ordered pairs (x_i, y_i) do not lie on a straight line. Find the equation of the straight line that best fits these data points. Also find the sixth data point.

2B. Find the coordinates of a vector $(2, 1)$ after rotating 90 degrees in counter clockwise direction, scaling by a factor of 2 in the x -direction and translate 3 units in the y -direction.

2C. Define the following and mention any one application: (i) Vandermonde matrix (ii) Permutation matrix.

(5+3+2)

3A. Find the Jordan canonical form of matrix A. Mention any two applications. $A = \begin{bmatrix} 1 & -3 & -2 \\ -1 & 1 & 1 \\ 2 & 4 & 5 \end{bmatrix}$

3B. How SVD is useful in detecting the edges in a colour image? Explain .

3C. Consider the matrix $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$. Which one of the following is an Eigen vector of matrix A? (i) $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$

(5+3+2)

4A. Solve the currents in the following electrical circuit using QR factorization.

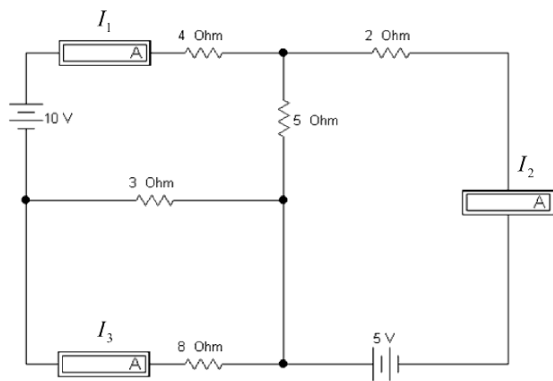


Figure 4A

4B. If the observed signal is $x(n) = Ae^{j\omega n} + \text{noise}$, and the autocorrelation matrix is estimated to be $\begin{bmatrix} 6 & 3 - j4 \\ 3 + j4 & 6 \end{bmatrix}$. Estimate the frequency of the signal and also find the noise variance.

4C. What are the benefits of Hermitian matrices?

(5+3+2)

5A. State Schwarz's inequality. Using this design a matched filter to detect a continuous signal in a digital communication system having a white noise channel with power spectral density $N_0/2$. Calculate the maximum SNR and the noise variance.

5B. What is pseudo inverse? Discuss the method of obtaining pseudoinverse of a system of linear equations.

5C. Find an equation involving g, h and k that makes this augmented matrix correspond to a consistent system.

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array} \right]$$

(5+3+2)