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# INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

## I SEMESTER B.S. DEGREE EXAMINATION – NOV. 2017

**SUBJECT: MATHEMATICS - I (MA 111)**

**(COMMON TO ALL BRANCH)**

**Monday, 13 November 2017**

**Time: 3 Hours** **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions.**
- ✓ **Missing data, if any, may be suitably assumed**

**1A.** If $y=\sin^{-1}x$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. Hence determine $y_n(0)$.

**1B.** Find the area common to the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2$.

**1C.** Obtain a reduction formula for $\int \cos^n x dx$ when n is a non-negative integer and evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x dx$. **(7 + 7 + 6)**

**2A.** Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$.

**2B.** Find the area of the cardioid $r = a(1 + \cos\theta)$.

**2C.** Evaluate $\int_0^a x\sqrt{ax-x^2}\, dx$. **(7 + 7 + 6)**

**3A.** Using Lagrange's mean value theorem, show that

$$\frac{v-u}{1+v^2} < \tan^{-1}v - \tan^{-1}u < \frac{v-u}{1+u^2},\quad 0<u<v.$$

Hence deduce that $\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$.

**3B.** If the curve $(a-x)y^2 = a^2x$ revolves about its asymptote, find the volume so formed.

**3C.** Trace the curve $y^2(a-x) = x^3, a>0$ with explanation.

**(7 + 7 + 6)**

**4A.** Expand $\log(1+\sin x)$ in powers of $x$ up to the terms containing $x^5$.

**4B.** Find the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

**4C.** Trace the curve $x = a\,(\theta + \sin\theta)\,, y = a\,(1 + cos\theta),\ a > 0$ with explanation.

**(7 + 7 + 6)**

**5A.** Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.

**5B.** Find the equation of the plane passing through (1, 0, -2) and perpendicular to the planes 2x + y – z = 2, x – y – z = 3.

**5C.** Trace the curve $r = a \sin 3\theta, a > 0$ with explanation.

**(7 + 7 + 6)**

**6A.** The tangents at two points P and Q on the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ are at right angles. If $\rho_1$ and $\rho_2$ are the radii of curvature at these points, then show that $\rho_1^2 + \rho_2^2 = 16a^2$.

**6B.** Find the equations of the planes bisecting the angle between the planes 7x + 4y + 4z + 3 =0, 2x + y + 2z + 2 = 0 and specify the one which bisects the acute angle.

**6C.** Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots \infty$.

**(7 + 7 + 6)**

**7A.** Find the $n^{th}$ derivative of the following functions:

(i) $\frac{x^2}{(x+2)(2x+3)}$ (ii) $\cos x \cos 2x \cos 3x$

**7B.** Find the shortest distance between the straight lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

**7C.** Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$.

**(7 + 7 + 6)**

**8A.** State the Cauchy's mean value theorem and verify for the functions sin x and cos x in (a, b), 0< a < b.

**8B.** Find the equation to a sphere which passes through the circle $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0,\ 2x + y + z = 4$ and through the point (1, 2, -1).

**8C.** Test the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} - \frac{3^3}{4^4} \ldots \infty$.

**(7 + 7 + 6)**