Reg.No.									

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

I SEMESTER B.Sc. (Applied Sciences) EXAMINATION- NOV. 2017

SUBJECT: MATHEMATICS - I (IMA 111)

Monday, 13 November 2017

Time: 3 Hours **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A. If $= e^{m\cos^{-1}x}$, prove that $(1-x^2)y_2 - xy_1 = m^2y$ and hence show that

$(1-x^2)y_{n+2} - \quad (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$

1B. Find the area between the curve $x(x^2+y^2) = a(x^2-y^2), a>0$ and its asymptote. Also find the area of its loop.

1C. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$ when n is a non-negative integer and evaluate

$\int_0^{\frac{\pi}{2}} \cos^4 x dx$.

(7+7+6)

2A. Show that the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $(ax)^{\frac{2}{3}} - (by)^{\frac{2}{3}} = (a^2+b^2)^{\frac{2}{3}}$.

2B. Find the area enclosed between one arc of cycloid $x = a(\theta - sin\theta)$,

$y = a(1-cos\theta), a > 0$ and its base.

2C. Evaluate $I_n = \int_0^a (a^2-x^2)dx$, where n is a positive integer. Hence show that $I_n = \frac{2n}{2n+1}a^2 I_{n-1}$.

(7+7+6)

3A. Show that $\frac{x}{1+x} < \log(1+x) < x, for\ all\ x > 0$. Hence show that $0 < [\log(1+x)]^{-1} < 1\ for\ all\ x > 0$.

3B. Find the volume of the solid obtained by revolving the Cissoid $y^2(2a-x)=x^3, a>0$ about its asymptote.

3C. Trace the parametric curve $x=a(t-\sin t)$ and $y=a(1+\cos t), a>0$ with explanation.

(7+7+6)

4A. Expand $tan^{-1}x$ in powers of $(x-1)$ upto terms containing $(x-1)^4$

4B. Find the entire length 'S' of the curve $x=a\cos^3\theta; y=a\sin^3\theta, a>0$.

4C. Trace $r^2 = a^2cos2\theta, a > 0$ with explanation.

(7+7+6)

5A. i) Find the angle between the curves $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$.

ii) Prove that $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$.

5B. From the following table, find the number of students who obtained marks between 40 and 45:

Marks obtained	30-40	40-50	50-60	60-70	70-80
No.of students	31	42	51	35	31

5C. Trace the curve $r = a(\,1 + sin\theta), a > 0$ with explanation.

(7+7+6)

6A. For the cardioid $r = a(1 + cos\theta), a > 0$ show that the square of the radius of curvature at any point (r,θ) is proportional to r. Also find the radius of curvature when $\theta = 0, \frac{\pi}{4}$.

6B. i) Derive the Newton's forward interpolation formula.

ii) With the suitable assumptions find the missing terms in the following table

X	1	2	3	4	5	6	7
y	103.4	97.6	122.9	-	179.0	-	195.8

6C. Test for the convergence of the series $\frac{2}{1}+\frac{2\cdot5}{1\cdot5}+\frac{2\cdot5\cdot8}{1\cdot5\cdot9}+\frac{2\cdot5\cdot8\cdot11}{1\cdot5\cdot9\cdot13}+\cdots$

(7+7+6)

7A. Find the n[th] derivative of $y=\frac{x^2}{(x-1)^2(x+2)}$

7B. Find the interpolating polynomial for the following data and hence find the value of f(9).

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

7C. Evaluate: (i) $\lim_{x\to o}\frac{e^x\sin x-x-x^2}{x^2+x\log(1-x)}$ (ii) Evaluate $\lim_{\theta\to\frac{\pi}{2}}\frac{\log(\theta-\frac{\pi}{2})}{tan\theta}$.

(7+7+6)

8A. Show that $\frac{v-u}{1+v^2}<tan^{-1}v-tan^{-1}u<\frac{v-u}{1+u^2}\ for\ 0<u<v$. Hence deduce $\frac{\pi}{4}+\frac{23}{5}<tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$.

8B. Find the equation of the sphere having the circle

$x^2+y^2+z^2+10y-4z-8=0,\ \ x+y+z=3$ as a great circle.

8C. Show that the series $\frac{x}{\sqrt{3}}-\frac{x^2}{\sqrt{5}}+\frac{x^3}{\sqrt{7}}-\cdots$ is absolutely convergent for

$-1<x<1$, Conditionally convergent for $x=1$.

(7+7 +6)