Reg. No.										

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

II SEMESTER B.S. DEGREE EXAMINATION – NOV. 2017

SUBJECT: MATHEMATICS -II (MA 121)

Tuesday, 14 November 2017

Time: 3 Hours **Max. Marks: 100**

- **Answer ANY FIVE full Questions.**
- **Draw diagrams and equations whenever necessary.**

1A.(i) Determine whether $\vec{F}$ is conservative. If so, find its scalar potential.

$$\vec{F} = (y^2 cosx + z^3)\boldsymbol{i} + (2ysinx - 4)\boldsymbol{j} + (3xz^2 + 2)\boldsymbol{k}$$

(ii) Let $\vec{A} = x^2z^2\boldsymbol{i} - 2y^2z^2\boldsymbol{j} + xy^2z\boldsymbol{k}$. Find $curl(curl\ \vec{A})$.

1B. Test for consistency and solve the following system of equations by using Gauss-elimination method.

$$x + y - 2z + 3w = 0$$
$$x - 2y + z - w = 0$$
$$4x + y - 5z + 8w = 0$$
$$5x - 7y + 2z - w = 0$$

1C. If $z = x^2 + y^2,\ x = \cos uv,\ y = \sin(u+v)$, find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ interms of u and v.

(8+8+4)

2A. If $u = cosec^{-1}\left[\frac{\sqrt{x}+\sqrt{y}}{\sqrt[3]{x}+\sqrt[3]{y}}\right]^{1/2}$ prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$.

2B.(i) Find the area common to the circles $r = a\sin\theta$ and $r = a\cos\theta$ by using double integrals.

(ii) Evaluate $\iint_R xy\ dxdy$, where R is the region bounded by $y^2 = 4x;\ y = 2x - 4$.

2C. Find the inverse of the following matrix by using Gauss-Jordan method $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

(8+8+4)

3A.(i) If $z = f(x,y)$ and $x = r\cos\theta,\ y = r\sin\theta,$ prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

(ii) Discuss the maxima and minima of the function

$$f(x,y) = sinx + siny + sin(x+y), 0 \le x,\ y \le \frac{\pi}{2}$$

3B. Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{x^2}{\sqrt{1+x^4}}dx = \frac{\pi}{4\sqrt{2}}$

3C. Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.

(8+8+4)

4A. (i) Let $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$. Evaluate $\iint_S \vec{F}.\vec{n}\,dS$ Where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

(ii) Evaluate $\iiint \nabla \cdot \vec{r}dr$, where $\vec{r}$ is a position vector.

4B. Define orthogonal and orthonormal set of vectors. Using Gram Schmidt process construct an orthonormal set of basis vectors of $V_3(R)$ for given vectors $a_1 = (1,-1,0),\ a_2 = (2,-1,-2),\ a_3 = (1,-1,-2)$

4C. By changing the order of integration evaluate $\int_0^{4a}\int_{\frac{x}{4a}}^{2\sqrt{ax}} dydx$

(8+8+4)

5A. (i) Verify Green's theorem in plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is a closed curve of the region bounded by $y = x$ and $y = x^2$.

(ii) If $z = e^{ax+by} f(ax - by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz.$

5B. By using double integrals, find the volume bounded by the cylinder $x^2 + y^2 = ay$ and the sphere $x^2 + y^2 + z^2 = a^2$

5C. If $z = f(x,y)$, and $x = e^u + e^{-v}, y = e^{-u} + e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\,\frac{\partial z}{\partial x} - y\,\frac{\partial z}{\partial y}$.

(8+8+4)

6A. If $z = x^m f\left(\frac{y}{x}\right) + x^n g\left(\frac{x}{y}\right)$ prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + mnz = (m+n-1)\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right)$$

6B. Evaluate the following integrals.

i) $\int_0^2 x\left(8-x^3\right)^{\frac{1}{3}}dx$ **ii)** $\int_5^7 (x-5)^6(7-x)^3\,dx$

6C. Find the volume inside the cone $x^2+y^2=z^2$ bounded by the sphere $x^2+y^2+z^2=a^2$ by converting to spherical polar co-ordinates.

(8+8+4)

7A.(i) If sides of a triangle ABC vary in such a way that its circum radius is constant prove that $\frac{\delta A}{\cos A}+\frac{\delta B}{\cos B}+\frac{\delta C}{\cos C}=0.$

(ii) Find $f(r)$ such that $f(r)\vec{r}$ is solenoidal.

7B. If R is the region bounded by $x=0,\ y=0$ and $x+y=1$, then evaluate the following double integrals by using transformations $x+y=u,\ x-y=v$.

i) $\iint_R \sin\left(\frac{x-y}{x+y}\right)dxdy$ **ii)** $\iint_R \cos\left(\frac{x-y}{x+y}\right)dxdy$

7C. Find the values of a and b such that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at P(1,-1,2)

(8+8+4)

8A.i) Find the dimensions of rectangular cube open at top, of maximum capacity whose surface area is 432 Sq.cm.

ii) If $u=e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x\partial y\partial z}$

8B. i) Test whether the set B={(1,1,0), (3,0,1), (5,2,2)} forms a basis for R^3. If so represent (1,2,3) in terms of basis vectors.

ii) Prove that the vectors $a_1,a_2,\ldots,a_m$ from E^n are linearly dependent if and only if one of the vectors is a linear combination of the other.

8C. Prove that $\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma(m+n)}$

(8+8+4)