Reg.No.									

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
(Manipal University)

THIRD SEMESTER B.S. DEGREE EXAMINATION – OCT. / NOV. 2017

SUBJECT: MATHEMATICS - III (MA 231)
(COMMON TO ALL BRANCH)
Monday, 30 October 2017

Time: 3 Hours **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A. Solve: $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$.

1B. Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$.

1C. Find the inverse Laplace transform of $\frac{s+2}{s^2-4s+13}$. **(7 + 7 + 6)**

2A. Solve: $xy(1+xy^2)\frac{dy}{dx} = 1$.

2B. Solve $\frac{dy}{dx} = x + y$ given $y(1) = 0$ and find y at x = 1.1 and x = 1.2 to five decimal places taking h = 0.1 by Taylor's series method (carry up to fourth order derivatives).

2C. Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+9)}\right]$. **(7 + 7 + 6)**

3A. Solve: $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$.

3B. Solve $\frac{dy}{dx} = y + e^x$, $y(0) = 0$ by Euler's modified method, for $x = 0.2$ and $x = 0.4$ taking h = 0.2. Carry out two iterations for each step.

3C. Solve $y'' - 2y' + y = e^t$, $y(0) = 2, y'(0) = -1$ using Laplace transform method. **(7 + 7 + 6)**

4A. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x sinx$.

4B. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$. Compute y at $x = 0.2$ and $x = 0.4$ by taking $h = 0.2$ using Runge - Kutta method of order four.

4C. Given that $f(z) = u + i\,v$ is analytic and $v(x,y) = -\sin x \sin hy$. Show that $v(x,y)$ is harmonic. Find the conjugate harmonic of $v(x,y)$.

(7 + 7 + 6)

5A. Find the Laplace transform of $f(t) = \cos^2 t + \sin 2t \sin 3t + \sin 3t \cos 4t$.

5B. Given that $f(z) = u + iv$ is analytic and $u(r,\theta) = \left(r - \frac{1}{r}\right)\sin\theta, r \neq 0$.

Find the orthogonal trajectories of the family of curves, $u = c$, a constant.

5C. Solve $\frac{d^2y}{dx^2} + a^2y = sec^2\, ax$ by using method of variation of parameters.

(7 + 7 + 6)

6A. Evaluate the integral $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt$ using Laplace transform.

6B. Solve: $(2x-1)^2\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} - 2y = 8x^2 - 2x + 3$.

6C. Evaluate $\oint_C \frac{dz}{z^2+9}$, where C is $|z - 3i| = 4$, using Cauchy's integral formula.

(7 + 7 + 6)

7A. Solve: $\frac{dx}{dt} + 2y = e^t$; $\frac{dy}{dt} - 2x = e^{-t}$.

7B. Find the Laplace transform of triangular wave of period $2a$ given by

$$f(t) = \begin{cases} t, & 0 < t \leq a \\ 2a - t, & a < t < 2a \end{cases}.$$

7C. Obtain the Taylor's series expansion of $f(z) = \frac{1-z}{z^2}$ in powers of $(z-1)$.

(7 + 7 + 6)

8A. Solve $u_{xx} + 2u_{xy} + u_{yy} = 0$ using the transformation $v = x,\ z = x - y$.

8B. Rewrite $f(t) = \begin{cases} t, & 0 \leq t < 2 \\ t^2 + 3, & 2 \leq t < 3 \\ 2 - t, & t \geq 3. \end{cases}$ in terms of unit step function and hence find its Laplace transform.

8C. Using Cauchy's Residue theorem, evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\,dz$, where C is the circle $|z| = 3$.

(7 + 7 + 6)