Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

III SEMESTER B.TECH (CS/IT/CC) END SEMESTER EXAMINATIONS, NOV 2017

SUBJECT: ENGINEERING MATHEMATICS III [MAT -2105]

REVISED CREDIT SYSTEM

(23/ 11/2017)

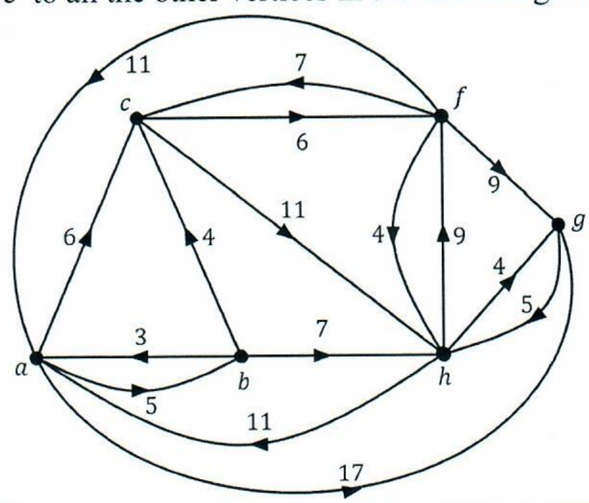
Time: 3 Hours

MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** questions.

1A.	Let $E(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\overline{x}_2 \wedge x_3)$ be a Boolean expression over the two-valued Boolean algebra. Express it in conjunctive normal form and disjunctive normal form.	**3**
1B.	How many permutations of $1, 2, \ldots, n$ are there in which the number k is not in k^{th} position for any k?	**3**
1C.	i) If G is a tree, then show that G is connected and the number of vertices of G is equal to one more than the number of edges in G. ii) Show that every self-complementary graph has $4n$ or $4n+1$ vertices for some positive integer n.	**4**
2A.	Let $\langle A, \vee, \wedge, \overline{\ \ } \rangle$ be a finite Boolean algebra. Let b be any nonzero element in A and $a_1, a_2, \ldots, a_k$ all the atoms of A such that $a_i \leq b$. Prove that $b = a_1 \vee a_2 \vee \cdots \vee a_k$.	**3**
2B.	Show that $S \vee R$ is tautologically implied by the premises $P \vee Q$, $(P \to Q)$, $(Q \to S)$.	**3**
2C.	Let $(G, *)$ be a group, $(H, *)$ a subgroup of G and x an element of G. Show that $x * H * x^{-1} = \{x * h * x^{-1} \mid h \in H\}$ is a normal subgroup of G if and only if $x * H * x^{-1} = H$.	**4**
3A.	Show that every cyclic group is Abelian and check whether a group on 83 elements is Abelian. Justify your answer.	**3**
3B.	Show that from $(\exists x)\,(F(x) \wedge S(x)) \to (y)(M(y) \to W(y))$ and $(\exists y)\,(M(y) \wedge \neg W(y))$ the conclusion $(x)\,(F(x) \to \neg S(x))$ follows.	**3**

3C.	(i) Show that in a distributive lattice, if an element has a complement, then the complement is unique. (ii) Let $(A,\leq)$ be a distributive lattice. Show that if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some element a of A, then $x = y$.	**4**
4A.	Let p_n be the number of unrestricted partitions of n, and let p_n^* be the number of partitions of n without unit parts. Show that for $n > 1$, $p_n^* = p_n - p_{n-1}$. Generalise this result to find a formula for the number of partitions of n without parts of size k.	**3**
4B.	Show that a non-empty subset H of a group G is a subgroup of G if and only if for all $a,b \in H$, $ab^{-1} \in H$.	**3**
4C.	Let $R(r,k)$ denote the number of partitions of the integer r into k parts. (i) Show that $R(r,k) = R(r-1,k-1) + R(r-k,k)$. (ii) Show that $\sum_{k=1}^{r} R(n-r,k) = R(n,r)$.	**4**
5A.	(i) Given $n = 5$ and the five marks $1,2,3,4,5$, what are the 50^{th} and 75^{th} permutations in the (a) Lexicographical ordering (b) Fike's ordering. (ii) Show that $a \vee (a \wedge b) = a$ for every a, b in the lattice $(L,\leq)$.	**3**
5B.	Show that in any party with six people there are either three people who mutually know each other or three people who mutually do not know each other.	**3**
5C.	Write the distance matrix for the following network. Using Dijkstra's algorithm, find shortest paths from c to all the other vertices in the network given below. 	**4**