Reg. No. ☐☐☐☐☐☐☐☐☐☐

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

INSPIRED BY LIFE

A Constituent Institution of Manipal University

III SEMESTER B.TECH. (Mech/ IP/Auto/Aero/MT)

END SEMESTER EXAMINATIONS, NOV 2017

SUBJECT: ENGINEERING MATHEMATICS III - MAT 2101

Time: 3 Hours

MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitable assumed.

1A.	Obtain the Fourier series for $f(x) = \begin{cases} x; & 0 \le x \le \pi \\ 2\pi - x; & \pi \le x \le 2\pi \end{cases}$ and hence deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$, given that $f(x + 2\pi) = f(x)$.	**04**
1B.	Solve $(x^3 + 1)y'' + x^2 y' - 4xy = 2$ Subjected to the conditions $y(0) = 0, y(2) = 4$ by taking h = 0.5.	**03**
1C.	Solve $\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2}$ $, 0 < x < 1,\ t > 0$ subjected to the conditions $u(x, 0) = 100\, sin\pi x, u\,(0,\ t\,) = u(1, t) = 0$. Compute u for one time step with h = 0.25 using Crank Nicolson's method	**03**
2A.	Classify and solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $0 < x < 1, t > 0$ with $u(x, 0) = 100(x - x^2)$, $\frac{\partial u}{\partial t}(x, 0) = 0, u\,(0, t) = u(1, t) = 0$ Choosing h = 0.25 for four time steps.	**04**
2B.	Solve $y'' + (1 + x)y' - y = 1$ Subjected to the conditions $y(0) = y'(0),\ y(1) + y'(1) = 1$ by taking h = 0.5	**03**

<table>
<tr><td>2C.</td><td>Find the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of y as given in the following table.
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td><td>9</td></tr>
</table></td><td>03</td></tr>
<tr><td>3A.</td><td>Determine whether $\vec{F} = (y^2 cosx + z^3)\hat{\imath} + (2y\, sinx - 4)\hat{\jmath} + (3xz^2 + 2)\hat{k}$ is conservative? If so find scalar potential.</td><td>03</td></tr>
<tr><td>3B.</td><td>From the Fourier integral show that $\int_0^\infty \frac{s \sin sx}{1+s^2}\, ds = \frac{\pi}{2} e^{-x} \ (x > 0)$</td><td>03</td></tr>
<tr><td>3C.</td><td>Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & \text{for } |x| < a \\ 0, & for\ |x| > a > 0 \end{cases}$ and deduce that $\int_0^\infty \left(\frac{sint}{t}\right)^4 dt = \frac{\pi}{3}$.</td><td>04</td></tr>
<tr><td>4A.</td><td>Derive the one dimensional heat equation with suitable assumptions.</td><td>04</td></tr>
<tr><td>4B.</td><td>Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$</td><td>03</td></tr>
<tr><td>4C.</td><td>Given $\vec{A} = (yz + 2x)\hat{\imath} + xz\hat{\jmath} + (xy + 2z)\hat{k}$. Evaluate $\int \vec{A}.d\vec{r}$ along the curve $x^2 + y^2 = 1$ & $z = 1$ in the positive direction of $(0, 1, 1)$ to $(1, 0, 1)$</td><td>03</td></tr>
<tr><td>5A.</td><td>Solve the partial differential equation $U_{xx} + U_{xy} - 2U_{yy} = 0$ using the transformation $v = x + y, z = 2x - y$</td><td>03</td></tr>
<tr><td>5B.</td><td>State and prove Green's theorem in the plane.</td><td>03</td></tr>
<tr><td>5C.</td><td>Verify divergence theorem for $\vec{A} = 4x\, i - 2y^2\, j + z^2\, k$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ & $z = 3$.</td><td>04</td></tr>
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