R.No.										

MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER M.Sc. (APPLIED MATHEMATICS AND COMPUTING) DEGREE
END SEMESTER EXAMINATION- NOV 2017
SUBJECT: ALGEBRA (MAT 4103

Time: 3 Hrs. 18 November, 2017 **Max. Marks: 50**

Note: a) Answer any FIVE full questions b) All questions carry equal marks

1A. Let H and K be subgroups of G. Prove that HK is a subgroup of G if and only if HK = KH.
1B. Define characteristic of an integral domain. Prove that the characteristic of an integral domain is either 0 or prime number. What is the Char (Z)?
1C. Let G be a finite group, and let p be prime. Prove that if $p^m \mid o(G)$ then G has a subgroup of order p^m.

(3 + 3+ 4)

2A. Prove that the set of all nilpotent elements in a commutative ring R with 1 is the intersection of all prime ideals.
2B. Prove that every cyclic group is isomorphic to Z or Z_n for some natural number n.
2C. Let N be a subgroup of a group G. Prove that the following are equivalent.

i. N is a normal subgroup of G.
ii. $xN = Nx$ for every $x \in G$.
iii. $(xN)(yN) = xyN$ for all $x, y \in G$. **(3 + 3 + 4)**

3A. Let R be a commutative ring with 1 and I be an ideal in R. R/I is an integral domain if and only if I is a prime ideal in R.
3B. Let f: $G \rightarrow G^1$ be a homomorphisms of groups. Then prove that G/Ker f $\cong$ Im f. In particular, if f is surjective, then G/Ker f $\cong G^1$.
3C. Prove that the set Aut (G) of all automorphisms of a group G is a group under composition of mappings, and In(G) is a normal subgroup of Aut(G).

(3 + 3+ 4)

4A. If G is a finite group, then prove that $|G| = \sum [G: N(a)]$, running over exactly one element from each conjugate class.
4B. Let G be a finite group of order p^n, where p is a prime number and $n > 0$. Then prove that G has a non-trivial center.
4C. Let G be a group. Define $a * x = ax$ for all $a, x \in G$. Verify whether G is a G-set. If X is a G-set then prove that the action of G on X induces a homomorphism $\phi: G \rightarrow S_x$.

(3 + 3+ 4)

5A. Find order of the given element in the direct product: (1, 4) in $Z_4 \times Z_{12}$; (2, 6) in $Z_6 \times Z_{15}$; (2, 4, 12, 16) in $Z_4 \times Z_{12} \times Z_{20} \times Z_{24}$.
5B. Let H be the internal direct product of its subgroups $H_1, H_2, \ldots, H_n$. Then show that H_i is a normal subgroup of H and $H_i \cap (H_1, H_2, \ldots H_{i-1}, H_{i+1}, \ldots, H_n) = \{e\}$ for all $1 \le i \le n$.
5C. Let G be a finite group and X a finite G-set. Prove that G_x is a subgroup of G. If r is the number of orbits in X under G, then prove that $r \cdot |G| = \sum_{g \in G} |X_g.|$ **(3 + 3 + 4)**