MANIPAL UNIVERSITY

Reg. No. | | | | | | | | | |

Deemed- to -be -University under Section 3 of the UGC Act, 1956

DEPARTMENT OF SCIENCES,
I SEMESTER M.Sc (Applied Mathematics & Computing)
END SEMESTER EXAMINATIONS, Nov/Dec 2017

Subject DIFFERENTIAL EQUATIONS (MAT-4101)

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date 16.11.2017 MAX. MARKS: 50

Note: (i) Answer all **FIVE FULL** questions **(3 + 3 + 4 Marks)**

(ii) Draw diagrams, and write equations wherever necessary

1A. If $\varphi_1(x)$ is a solution of a differential equation $y'' + a_1(x)\,y' + a_2(x)\,y = 0$ then show that $\varphi_2(x) = \varphi_1(x)\,f(x)$ is a solution of this equation provided $f'(x)$ satisfies the equation $(\varphi_1^2\, y)' + a_1(x)\,(\varphi_1^2\, y) = 0$

1B .Let φ_1, φ_2 be be two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$. Then prove that φ_1, φ_2 are linearly independent on an interval I if and only if $W(\varphi_1, \varphi_2)(x) \neq 0$.

1C If $\varphi_1, \varphi_2, \ldots\ldots \Phi_n$, are n solutions of $L(y) = 0$ on an interval I containing a point x_0 then prove that $W(x) = e^{-a_1(x-x_0)}\,W(x_0)$

2A. Starting from generating Bessel function derive Jacobi series.

2B. Obtain the Rodrigue's formula $P_n(x) = \dfrac{1}{n!\,2^n}\dfrac{d^n}{dx^n}(x^2-1)^n$ where $P_n(x)$ is Legendre polynomial of degree n.

2C. State and prove uniqueness theorem for Let $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \cdots\cdots + a_n y = 0$ on an interval I containing a point x_0. With usual initial conditions

3A. Find a function $\varphi(x)$ which has a continuous derivative on $0 \leq x \leq 2$ which satisfies $\varphi(0) = 0$, $\varphi'(0) = 1$ and $y'' - y = 0$ for $0 \leq x \leq 1$ and $y'' - 9y = 0$ for $1 \leq x \leq 2$.

3B Prove that (i) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{3 - x^2}{x^2} \sin x - \frac{3}{x} \cos x \right\}$

(ii) $J_n''(x) = \frac{1}{4}[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)]$

3C. Prove that $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \begin{cases} 0 & , \alpha \neq \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2 & , \alpha = \beta \end{cases}$ where α, β are the roots of $J_n(x) = 0$.

4A Test the solution functions φ_1, φ_2, defined by $\varphi_1 = x^2$, $\varphi_2 = x\,|x|$, for linearly independent on $-\infty < x < \infty$.

4B. . Suppose ϕ is a function having continuous derivative on $0 \leq x < \infty$ such that $\phi'(x) + 2\phi(x) \leq 1$ for all x and $\phi(0) = 0$. Show that $\phi(x) \leq \frac{1}{2}$ for $x \geq 0$.

4C Prove that (1) $H_n'(x) = 2n\, H_{n-1}(x)$

(2) $2x\, H_n(x) = 2n\, H_{n-1}(x) + H_{n+1}(x)$

5A. Solve by method of variation of parameters $y''' + y'' + y' + y = 1$

5B Show that $\int_{-1}^{1} x^2\, P_{n-1}(x)\, P_{n+1}(x)\, dx = \frac{2n\,(n+1)}{(2n-1)(2n+1)(2n+3)}$

5C. Obtain the series solution of $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)\,y = 0$, n is a real number.