MANIPAL UNIVERSITY

Reg. No.										

Deemed- to -be -University under Section 3 of the UGC Act, 1956

DEPARTMENT OF SCIENCES
I SEMESTER M.Sc (Applied Mathematics & Computing)
END SEMESTER EXAMINATIONS, November - 2017

Subject [MAT 4105 - Real Analysis]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 21/11/2017 MAX. MARKS: 50

Note: (i) Answer all **FIVE FULL** questions

(ii) All questions carry equal Marks **(3 + 3 + 4)**

1A. Let $\{E_n\}$, n = 1, 2, 3, … , be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$. Show that S is countable.

1B. Define a compact set. If F is a closed subset and K is compact subset of a metric space X, show that $F \cap K$ is compact.

1C. Show that the set of real numbers R has Archimedean property and hence show that there exists a rational number between any two real numbers.

2A. Show that a set E is open if and only if its complement is closed.

2B. Define a Cauchy sequence and a complete metric space. In any metric space X, show that every convergent sequence is a Cauchy sequence.

2C. Show that a mapping of a metric space X into a metric space Y is continuous on X if & only if $f^{-1}(V)$ is open in X for every open set V in Y.

3A. Obtain the circle of convergence and radius of convergence for the series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} z^{2n}$.

3B. Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then show that f(X) is compact.

3C. If p > 0, then show that $\lim_{n \to \infty} \sqrt[n]{p} = 1$.

4A. If P^* is a refinement of a partition P then show that $L(P, f, \alpha) \le L(P^*, f, \alpha)$.

4B. Let [a, b] be a given interval. Suppose that f is a bounded function on [a, b] and α is a monotonically increasing function on [a, b]. Show that $\int_{-a}^{b} f d\alpha \le \int_{a}^{-b} f d\alpha$.

4C. If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on [a, b] then show that $f_1 + f_2 \in R(\alpha)$ and that $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.

5A. Suppose K is compact, and

(i) $\{f_n\}$ is a sequence of continuous functions on K,
(ii) $\{f_n\}$ converges pointwise to a continuous function f on K,
(iii) $f_n(x) \ge f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3, …..

Then show that $f_n \to f$ uniformly on K.

5B. State and prove Cauchy criterion for uniform convergence of a sequence of functions $\{f_n\}$ defined on a set E.

5C. If X is a complete metric space, and if ϕ is a contraction of X into X, then show that there exists one and only one $x \in X$ such that $\phi(x) = x$.

****