MANIPAL UNIVERSITY

Deemed- to -be -University under Section 3 of the UGC Act, 1956

Reg. No.									

DEPARTMENT OF SCIENCES, M.Sc in Applied Mathematics &Computing
III SEMESTER MAKEUP EXAMINATION, DECEMBER 2017

SUBJECT: GRAPH THEORY [MAT 707]

(REVISED CREDIT SYSTEM)

Time: 3 Hours MAX. MARKS: 50

Note: (i) **Answer any FIVE full questions** (ii) **All questions carry equal marks (4+3+3).**

1. (a) Show that a graph G with $p \geq 3$, is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.

   (b) With usual notation, prove that $\alpha_0 + \beta_0 = \alpha_1 + \beta_1 = p$.

   (c) Let G_1be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Derive expressions for number of edges in $G_1 \times G_2$ and $G_1[G_2]$.

2. (a) Show that, for any connected graph G, if $diam(G) \geq 3$, then $diam(\bar{G}) \leq 3$. And hence show that diameter of a self-complementary graph is either 2 or 3.

   (b) Show that the Ramsey number $r(m, n)$ satisfies the in equality $r(n, n) \geq 2^{\frac{n}{2}}$.

   (c) Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

3. (a) Show that in a critical graph no vertex cut is a clique. Hence show that every critical graph is a block.

   (b) Let $(S_1, S_2, \dots, S_n)$ be any partition of the set of integers $\{1,2, \dots, r_n\}$ where $r_n = r(k_1, k_2, .., k_n)$ with $k_i = 3$ for every i. Then show that, for every i, $S_i$contains three integers x, y and z satisfying $x + y = z$.

   (c) Show that the maximum number of edges in graphs among all the p vertex graphs with no triangle is $\left[\frac{p^2}{4}\right]$.

4. a) Find the minimum integer n such that in any party with n persons, either there are 3 persons who mutually know each other or 4 persons who mutually do not know each other.

b) With usual notation show that $\mathrm{r}(3,3) = 6$.

c) Show that a graph is a line graph of a tree if and only if it is a connected block graph in which each cut vertex is on exactly two blocks.

5. a) Let G be a tree on p vertices and $Q(G)$ be the $(0,1,-1)$ incidence matrix of G. Then show that any submatrix of $Q(G)$ of order $(n-1)$ is nonsingular.

b) Show that a (p,q)graph G is a tree if and only if $p = q + 1$ and G is connected.

c) With usual notation show that $k(G) \leq \lambda(G) \leq \delta(G)$. .

6. a) If G is a (p,q) graph such that $\delta(G) \geq \frac{p}{2}$ then show that G is Hamiltonian.

(b) For any (p,q) graph G with line graph $L(G)$, show that $A\big(L(G)\big) = B^T B - 2I_q$ where B is the incidence matrix of G and $A(L(G))$ is the adjacency matrix of $L(G)$.

(c) Show that if G is a tree on p vertices, then $\pi_k(G) = k(k-1)^{p-1}$.

****