MANIPAL UNIVERSITY

Deemed- to -be -University under Section 3 of the UGC Act, 1956

Reg. No.									

DEPARTMENT OF SCIENCES, M.Sc in Applied Mathematics &Computing
III END SEMESTER EXAMINATION, NOVEMBER 2017

SUBJECT: **GRAPH THEORY** [MAT 707]

(REVISED CREDIT SYSTEM)

Time: 3 Hours MAX. MARKS: 50

Note: (i) **Answer any FIVE full questions** (ii) **All questions carry equal marks (4+3+3).**

1. (a) With usual notation show that

$$\text{(i) } 2\sqrt{p} \le \chi + \bar{\chi} \le p + 1$$

$$(ii)\ p \le \chi\bar{\chi} \le \left(\frac{p+1}{2}\right)^2.$$

(b) Show that a graph G is 2-connected if and only if between every two vertices of G there are vertex disjoint paths.

(c) Show that if a regular connected graph G is of diameter 3 then $\bar{G}$ is of diameter 2. Hence show that there does not exist a regular self-complementary graph of diameter 3.

2. (a) With usual notation, prove that $\alpha_0 + \beta_0 = \alpha_1 + \beta_1 = p$, for a non trivial connected (p, q) graph G.

(b) Let G be a connected (p, q) graph with $p \ge 3$. Then show that $\omega(G) = q$ if and only if G has no triangles.

(c) Show that the Ramsey number $R(m, n)$ satisfies the in equality $r(m, n) \le r(m-1, n) + r(m, n-1)$.

3. (a) Let G be any graph and let u and v be vertices in G such that $deg(u) + deg(v) \ge r$. Then show that G is Hamiltonian if and only if $G + (u, v)$ is Hamiltonian.

(b) For any (p, q) graph G with line graph $L(G)$, show that $A\big(L(G)\big) = B^T B - 2I_q$ where B is the incidence matrix of G and $A(L(G))$ is the adjacency matrix of $L(G)$. .

(c) Show that the maximum number of edges in graphs among all the p vertex graphs with no triangle is $\left[\frac{p^2}{4}\right]$.

4. a) Show that rank of incidence matrix B is $n-1$, when G has n vertices.

b) Show that in a critical graph, no vertex cut is a clique and hence show that every critical graph is a block.

c) With the usual notation show that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$.

5. a) If G is a k-critical graph then show that $\delta(G) \geq k-1$.

b) Let G be a bipartite graph with bipartition (X, Y). Then show that G contains a matching that saturates every vertex of X if and only if $|N(S)| \geq |S|$ for all

$S \subseteq$ X.

c) Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

6. a) Let G_1be a (p_1, q_1) and G_2be a (p_2, q_2) graphs. Then find number of vertices and edges in (i) $G_1 \times G_2$ (ii) $G_1[G_2]$.

b) Let G be a graph on n vertices. Columns $j_1, j_2, \dots, j_k$ of $Q(G)$ are linearly independent if and only if the corresponding edges of G induce an acyclic graph. Hence show that if G be a tree on n vertices, then any submatrix of $Q(G)$ of order $n-1$ is nonsingular.

c) Show that every planar graph is five colorable.

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