MANIPAL UNIVERSITY

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DEPARTMENT OF SCIENCES,
I SEMESTER M.Sc (Applied Mathematics & Computations)
END SEMESTER EXAMINATIONS, Nov/Dec 2017

SUBJECT : NUMERICAL ANALYSIS – II - MAT - 705

(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Date:20/11/2017** **Max.Marks : 50**

Note : a) Answer any FIVE full questions. b) All questions carry equal marks.

1A. Show that the error in the explicit formula for one dimensional wave equation is least when $r = 1$, where $r = (kc/h)$ is the mesh ratio parameter

1B. Using Heun's method find y for x =0.2 given that $y' + 2xy^2 = 0$, $y(0)=1$ and hence find the magnitude of the error if the exact solution is $y(x) = 1/(1+x^2)$

1C. (i) Derive the standard five point formula for Poisson equation and hence derive the truncation error for the same

(ii) In which part of the (x,y) plane is the following equation elliptic?

$U_{xx} + 4u_{xy} + (x^2 + 4y^2) U_{yy} = \sin(xy)$

(3 + 3 + 4)

2A. Prove that Schmidt method is convergent when $\lambda \le (1/2)$, where λ is the mesh ratio parameter.

2B. Solve the wave equation $u_{tt} = u_{xx}$, $0 \le x \le 1$, subject to the initial and boundary conditions : $u(x, 0) = \sin \pi x$, $u_t(x,0) = 0$, $0 \le x \le 1$ and $u(0, t) = 0$ and $u(1, t) = 0$, $t > 0$. Using the explicit method, compute u for four time levels with $h = 0.25$ and $k = 0.25$.

2C. Solve the initial value problem

$$\begin{aligned} y' &= -3y + 2z = 0 \ , \ y(0)=0 \\ z' &= 3y - 4z = 0 \ , \ z(0)=0.5 \end{aligned}$$

on the interval [0, 0.2], using R-K fourth order method.

(3 + 3 + 4)

3A. Obtain the truncation error relation in Simpson's 1/3 rule.

3B. Find the solution of $\nabla^2 u = 0$ in R subject to the Dirichlet condition $u(x,y) = x - y$ on ∂R , where R is the region inside the triangle with vertices (0,0), (7, 0) and (0,7) and ∂R in its boundary. Assume the step length $h = 0.2$.

3C. Using Milne's predictor-corrector method, find y(0.4) for the initial value problem $y' = x^2 + y^2$, $y(0)=1$ with h =0.1 . Calculate all the required initial value by Euler's method. The result is to be accurate for three decimal places.

(3 + 3 + 4)

4A. Obtain the test equation for initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ nd applying Euler's method to test equation determine its stability zone

4B. (i) State the sufficient condition for the existence of unique solution of the initial value problem

(ii) Derive the DuFort-Frankel method for solving one dimensional heat conduction equation.

4C. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -1$, $0 < |x| < 1$, $0 < |y| < 1$, under the boundary conditions $u(\pm 1, y) = u(x, \pm 1) = 0$, using Liebmann's method with h = 1/2.

(3 + 3 + 4)

5A. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions $u(x,0) = 100(x - x^2)$, $0 \le x \le 1$ $u(0, t) = 0$ and $u(1,t) = 0$ by Bender-Schmidt method, with h = 0.25 and k =1 . Compute for four time steps.

5B. Using second order finite difference method, solve the differential equation $y'' + (1+x)y' - y = 0$ and subject to the conditions $y(0) - y'(0) = 0$and $y'(1)+y(1) = 1$ with h = 0.5 using Gauss elimination method.

5C. Derive Adams-Moulton method and hence find the general term of truncation error.

(3 + 3 + 4)

6A. Use Crank-Nicolson method and the central differences for the boundary conditions to solve the initial boundary value problem $u_{xx} = u_t$ subject to the conditions $u(x,0) = 0$, $0 \le x \le 1$, $u(0,t) = 0$, $u(1,t) = t$ with step size h = 1/3 and λ = 1/3. Integrate up to one time level.

6B. Solve the following BVP using shooting method :

$2\,y\,y'' - y'^2 + 4\,y^2 = 0$, $y(\frac{\pi}{6}) = \frac{1}{4}$, $y(\frac{\pi}{2}) = 1$. Take α_0=0.5, α_1=0.8 as initial approximations to $y'(\frac{\pi}{6})$ and iterate until the condition at $x = \frac{\pi}{2}$ is satisfied to five places.

6C. Derive R-K method of order two and hence find the truncation error of the same.

(3 + 3 + 4)

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