

DEPARTMENT OF SCIENCES, III SEMESTER M.Sc.
APPLIED MATHEMATICS & COMPUTING
END SEMESTER EXAMINATIONS, Nov. 2017
Subject : OPTIMIZATION METHODS-I (MAT 701)

(REVISED CREDIT SYSTEM)

Time: 3 Hours

Date: 15-11-2017

MAX. MARKS: 50

Note: Answer any **FIVE FULL** questions

- 1A** A caterer must prepare from 5 fruit drinks in the stock 500 gal of punch containing at least 20% of orange juice, 10% of grape juice and 5% of cranberry juice. If the inventory data are as shown below, how much of each fruit drink should the caterer use to obtain the required composition at minimum total cost?

Drinks	Orange Juice, %	Grape juice, %	Cranberry Juice, %	Supply gal.	Cost \$/gal.
A	40	40	0	200	1.50
B	5	10	20	400	0.75
C	100	0	0	100	2.00
D	0	100	0	50	1.75
E	0	0	0	800	0.25

Formulate the problem mathematically.

- 1B.** A company manufactures two products A & B. It is estimated that the sales of product A for the next month will not exceed 20 units, whereas the company has to supply a minimum of 10 units of product B. There are 100 hours of machining possible and both the products requiring 4 hours machining each. To assemble the parts, 4 and 6 labour hours are required for products A & B and a total of 180 hours of labour is available. The material available with the company is sufficient to produce 40 units of both parts with both parts requiring the same amount of material. If profits obtained in selling one unit of products A & B are Rs. 30 and Rs.40, find the best production using graphical method.
- 1C.** Show that a basic feasible solution of a LPP in the standard form is the extreme point of the feasible solution space of the LPP. (3+4+3)
- 2A.** A furniture producer makes 3 types of Chairs A, B and C. He uses 3 machines to produce chairs. The time required (In hours) to produce each chair on 3 machines are given below

Chairs	Machines		
	I	II	III
A	2	4	3
B	3	3	2
C	2	1	4

During the next week there will be 120 hours available for machine I, 160 hours for machine II and 100 hours for machine III for production. The producer can sell a maximum of 40 chairs in the week. The profits in selling the chairs of type A, B, C will be Rs. 40, Rs. 35 and Rs. 30 respectively. Determine the best production of chairs using simplex method.

- 2B. The demand and production costs vary from month to month in an industry. The following table contains the budgeted information of a firm in the industry on the quantity demanded, the production cost per unit and the production capacity in each of the coming five months.

Month	January	February	March	April	May
Demand	200	250	150	80	120
Capacity	250	225	250	200	225
Production cost	24	27	32	30	34

It is known that the production in any month can meet demand in that month or can be held for the future with a holding cost of Rs.5 per unit per month. How should the manufacturer produce in each month to minimize total cost? (5+5)

- 3A. Solve the following LPP by branch and bound method

$$\text{Minimize } Z = 3x_1 + 2.5x_2$$

Subject to

$$x_1 + 2x_2 \geq 20,$$

$$3x_1 + 2x_2 \geq 50$$

$$0 \leq x_1, x_2 \text{ and integers.}$$

- 3B. The captain of Cricket team has to allot five middle batting positions to five batsman. The average runs scored by each batsman at these positions are as follows

		Batting Positions				
		I	II	III	IV	V
Batsmen	A	40	40	35	25	50
	B	42	30	16	25	27
	C	50	48	40	60	50
	D	20	19	20	18	25
	E	58	60	59	55	53

- (i) Find the assignment to batsmen to positions.

- (ii) If another batsman F with the following average runs in following positions as given below

Batting Position :	I	II	III	IV	V
Average runs :	45	52	38	50	49

is to be added to the team. Should he be included to play in the team? If so, who will be replaced by him?
(6+4)

4A. Reduce the game defined by the following pay off matrix into a 2 X 2 game and then solve it

		Player B			
		B ₁	B ₂	B ₃	B ₄
Player A	A ₁	3	1	3	2
	A ₂	2	7	-5	1
	A ₃	3	4	-1	2
	A ₄	3	3	-2	2

4B. A Project has following activities, precedence relations and duration.

Activities	A	B	C	D	E	F	G	H	I	J	K
Immediate predecessor	---	---	---	---	A	A	B	B	C,E	D,G	H
Duration (in days)	8	2	6	12	5	9	3	7	4	10	2

Activities	L	M	N	O	P	Q	R	S	T	U	V
Immediate predecessor	H	F,K	L	L	N	I,M,O	I,M,O	J	J	P,Q,S	U
Duration (in days)	10	12	3	6	8	18	9	7	4	11	4

Draw a network, find the critical path, duration of the critical path. Find total and free floats for all activities. If the activity 'O' is delayed by 5 days, will it change duration of the project? By how many days?
(4+6)

5A. A project has following activities precedence relations and time estimates

Activity	Immediate predecessor	Duration (in weeks)		
		Optimistic	Most likely	Pessimistic
A	-	2	3	4
B	-	3	4	11
C	-	2	5	8
D	A,B	1.5	3.5	8.5
E	B,C	5	7	9
F	A,B	2	5.5	6
G	C	1.5	2.5	6.5
H	D,E,F	3	4	11
I	D	4	6	8
J	G	3	4.5	9
K	G	5	6	7

L	H,J	1	3	11
M	K	4	5	6
N	I,L	6	7	8

Draw a network, find the critical path and expected duration of the project. In how many weeks the project will be completed with the probability of 0.95? Given $\phi^{-1}(0.95) = 1.645$

5B. Solve the game defined by the following payoff matrix by simplex method

		Player B		
		B ₁	B ₂	B ₃
Player A	A ₁	-1	-3	1
	A ₂	1	3	-5
	A ₃	4	0	-4

(5+5)

6A. Given a LPP, Maximize $Z = 4x - 2y - z$

$$\begin{aligned} \text{Subject to } & x + y + z \leq 3 \\ & 2x + 2y + z \leq 4 \\ & x - y \leq 0 \\ & x, y, z \geq 0 \end{aligned}$$

Solve the LPP by Simplex method and discuss the effect of change in availability of resources without change in the optimal value.

6B. A project has following activities, time and cost estimates.

Activity	Preceding Activities	Time (in weeks)		Cost (Rs.)	
		Normal	Crash	Normal	Crash
A	-	4	2	800	900
B	-	6	3	600	900
C	B	5	2	400	580
D	B	4	2	400	600
E	A	3	1	300	400
F	A	4	4	150	150
G	F	2	1	120	200
H	C,E,G	6	4	350	450
I	F	2	1	700	800
J	D	4	2	400	600

Indirect costs are Rs 70 per ^{week} day. Draw a network and find the critical path. Find the project duration which will minimize the total cost.

(5+5)