MANIPAL ACADEMY OF HIGHER EDUCATION
(Deemed-to-be-University under Section 3 of the UGC Act, 1956)

Reg. No. | | | | | | | | | | |

DEPARTMENT OF SCIENCES, M. Sc. (Physics)
I SEMESTER END SEMESTER EXAMINATIONS
DECEMBER 2017
Subject: Quantum Mechanics I (PHY-4105)
(REVISED CREDIT SYSTEM)

Time: 3 Hours   Date: December 2017   MAX. MARKS: 50

Note: (i) Answer all the questions.
(ii) Answer the questions to the point.

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1. (i) Describe a phenomena which shows particle nature of light. [5]

(ii) A representation is given by the base vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Construct the transformation matrix U for transformation to another representation consisting of basis vectors $\begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$ and $\begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$. Also show that the matrix is unitary. [5]

2. (i) Obtain expressions of energy eigenvalues and eigenfunctions for a particle in an infinite deep potential well. [5]
(ii) An electron in a one-dimensional infinite deep potential well, defined by $V(x) = 0$ for $-a \leq x \geq a$ and $V(x) = \infty$ otherwise, goes from the $n = 4$ to the $n = 2$ level. The frequency of the emitted photon is 3.43×10^{14} Hz. Find the width of the box. [5]

3. (i) Establish the Schroedinger equation for a SHO and reduce it into the Hermite equation. [5]
(ii) Calculate the number of revolutions per second which a rigid diatomic molecule makes when it is in the (i) $l = 2$ state, (ii) $l = 5$ state, given that the moment of inertia of the molecule is I. [5]

4. Write the Schroedinger equation for spherically symmetric potentials. Break it into r, θ, ϕ dependent equations using separation of variables technique. [10]

5. (i) Describe how symmetric and antisymmetric wavefunctions are constructed for a system of n identical particles? [5]

(ii) What is the ground state energy and wavefunction for two identical particles in an infinite deep potential of width a, if the two particles are (a) bosons, and (b) fermions? [5]

Useful formulae:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

$$\int_0^\infty exp(-a^2x^2)cos(bx)\,dx = \frac{\sqrt{\pi}}{2a}exp\left(-\frac{b^2}{4a^2}\right)$$

$$\int_0^\infty x^n exp(-ax)\,dx = \frac{n!}{a^{n+1}}, \quad \textbf{where} \quad n \geq 0, \quad a > 0$$

For hydrogen atom

$$\psi_{100} = \sqrt{\frac{1}{\pi a_0^3}}e^{-r/a_0}$$

where a_0 is the Bohr radius.