Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

A Constituent Institution of Manipal University

I SEMESTER M.TECH. (AUTOMOBILE ENGINEERING)

END SEMESTER EXAMINATIONS, NOVEMBER- 2017

SUBJECT: COMPUTATIONAL METHODS [MAT 5103]

REVISED CREDIT SYSTEM

Time: 3 Hours (28/11/2017) MAX. MARKS: 50

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A.	Use SOR method with $\omega = 1.25$ to solve $\begin{aligned} 4x_1 + 3x_2 &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= -24 \end{aligned}$. Carryout 3 iterations using $X^{(0)} = (1\;1\;1)^T$ and accurate to 4 decimal places.	**3 Marks**
1B.	Solve by Newton-Raphson method: $2x^2 + 3xy + y^2 = 3,\;\; 4x^2 + 2xy + y^2 = 30$. Start with the initial approximation $x_0 = -3,\; y_0 = 2$.	**3 Marks**
1C.	Solve the system of equations using Thomas method: $2x_1 - x_2 = 6$ $-x_1 + 3x_2 - 2x_3 = 1$ $-2x_2 + 4x_3 - 3x_4 = -2$ $-3x_3 + 5x_4 = 1$	**4 Marks**
2A.	Given $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$, $y(0) = 1$; find $y(0.06)$ with $h = 0.02$ by Euler's modified method.	**3 Marks**
2B.	Evaluate $y(0.1)$ by Taylor's series method, given $y' = xy + 1,\; y(0) = 1$.	**3 Marks**

2C.	Solve the differential equations $\frac{dy}{dx}=1+xz,\ \ \frac{dz}{dx}=-xy$ for $x=0.3$ using R K method. Take the initial values are $x=0, y=0, z=1.$	**4 Marks**
3A.	Solve $y''+xy=1,\ y(0)=0,\ y'(1)=1$ with $h=0.5$ by finite difference method.	**3 Marks**
3B.	Solve $u_{tt}=u_{xx},\ 0<x<1,\ t>0,\ u(x,0)=u_t(x,0)=0, u(0,\ t)=0,$ $u(1,\ t)=100\sin \pi t.$ Compute u for 4-time step by with $h=1/4.$	**3 Marks**
3C.	With $h=1$ and $u=0$ on the boundary, solve the Poisson's equation $u_{xx}+u_{yy}=-10(x^2+y^2+10),\ 0<x<3,\ 0<y<3.$	**4 Marks**
4A.	Use Galerkin's method to solve the boundary value problem $y''=3x+4y,\ 0<x<1,\ y(0)=0, y(1)=1.$	**3 Marks**
4B.	Apply Rayleigh-Ritz method to solve $y''+y+x=0 \quad (0\le x\le 1),$ $y(0)=y(1)=0.$ (Use one parameter approximate solution).	**3 Marks**
4C.	Solve the heat equation $u_t=u_{xx}$ by Bendre-Schmidt method, subject to the conditions $u(0,t)=u(1,t)=0$ and $u(x,0)=\begin{cases} 2x & \text{for } 0\le x\le 1/2 \\ 2(1-x) & \text{for } 1/2\le x\le 1 \end{cases}.$ Take $h=1/4$ and $k=1/96$, compute u for 4-time steps.	**4 Marks**
5A.	Derive mathematical model for population growth and effects of immigration and emigration on population size.	**3 Marks**
5B.	Discuss the 12 point procedure and classification of mathematical modelling.	**3 Marks**
5C.	Solve $u_t=u_{xx}+u_{yy}$ satisfying the initial condition: $u(x,y,0)=\sin \pi x \sin \pi y,\ 0\le x,y\le 1$ and boundary conditions $u=0$ at $x=0$ and $x=1$ for $t>0.$ Obtain the solution upto two time levels with $h=1/3,\ \alpha=1/8.$	**4 Marks**