Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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A Constituent Institution of Manipal University

I SEMESTER M.TECH. (BIO-MEDICAL/DEAC/MICRO-ELECTRONICS ENGINEERING)

END SEMESTER EXAMINATIONS, NOVEMBER 2017

SUBJECT: PROBABILITY, RANDOM VARIABLES & STOCHASTIC PROCESS [MAT 5104]

REVISED CREDIT SYSTEM

(28/11/2017)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- **Answer ALL the questions. Use of statistical tables is allowed.**

1A.	Suppose 2% of cotton fabric rolls and 3% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturer, 70% are cotton and 30% are nylon. What is the probability that a randomly selected roll used by the manufacturer contains flaws?	3 marks
1B.	Test coverage in semiconductor testing is assumed to be 80% effective, i.e., the probability that a defective chip fails the test is 0.8. Three defective chips are to be tested. Assume the failure of each defective chip is independent of the other tests. Let X be the random variable denoting the number of defective chips that fail the test. What is the expected value of X?	3 marks
1C.	Given $f(x,y)=xe^{-x(y+1)}, x>0, y>0$. Find the regression curves of Y on X , and X on Y.	4 marks
2A.	The sample values from a population with probability density function $f(x;\theta)=(1+\theta)x^{\theta}, 0<x<1, \theta>0,$ are as follows. 0.46, 0.38, 0.61, 0.82, 0.59, 0.53, 0.72, 0.44, 0.59, 0.60 Find the estimate of the parameter θ by the method of maximum likelihood estimation.	3 marks
2B.	The process of drilling holes in printed circuit boards produces diameters with a standard deviation of 0.01 millimeter. How many diameters must be measured so that the probability is at least 8/9 that the average of the measured diameters is within 0.005 of the process mean diameter μ ?	3 marks
2C.	When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows. No. of mistakes/page: 0 | 1 | 2 | 3 | 4 | 5 | 6 No. of pages: 275 | 72 | 30 | 7 | 5 | 2 | 1 Fit a Poisson distribution to the given data and test the goodness of fit at 5% level of significance.	4 marks
3A.	Let p be the probability that a coin will fall head in a single toss in order to test $H_0: p=0.5 vs H_1: p=0.75$. The coin is tossed five times and H_0 is rejected if more than three heads are obtained. Find the size and power of the test.	3 marks
3B.	The thickness of photoresist applied to wafers in semiconductor manufacturing at a particular location on the wafer is uniformly distributed between 0.2050 and 0.2150 micrometers. Determine the cumulative distribution function of photoresist thickness. Also determine the proportion of wafers that exceed 0.2125 micrometers in photoresist thickness.	3 marks

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3C. An experiment was performed to determine the effect of four different chemicals on the strength of a fabric. These chemicals are used as part of the permanent press finishing process. Five fabric samples were selected, and a randomized complete block design was run by testing each chemical type once in random order on each fabric sample. The data are shown below. Test for differences in means using the analysis of variance at 1% level of significance. — 4 marks

Chemical type	Fabric sample				
	1	**2**	**3**	**4**	**5**
1	1.3	1.6	0.5	1.2	1.1
2	2.2	2.4	0.4	2.0	1.8
3	1.8	1.7	0.6	1.5	1.3
4	3.9	4.4	2.0	4.1	3.4

4A. Find the probability generating function of a binomial random variate with parameters n and p. Hence obtain its mean and variance. — 3 marks

4B. Let $\{X(t)\}$ be a stochastic process defined by $X(t)=\cos(t+\phi), \phi$ is uniform in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right), t$ and ϕ are independent of each other. Verify whether $\{X(t)\}$ is stationary. — 3 marks

4C. Transform the following process into a Markov chain and find its transition probability matrix.
Suppose it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it has rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.
Find the period of the Markov chain. Also examine the nature of its states. — 4 marks

5A. Machines fail at 4 per hour and the cost of non-productive machine is Rs 200/- per hour. A repairman charges Rs. 100/- per hour and repairs at 5 per hour. What will be the total queuing costs per hour? (Assume M/M/1 queueing system) — 3 marks

5B. An overhead crane of ABC Ltd moves jobs from one machine to another and must be used every time a machine requires loading or unloading. The demand for service is random. Data taken by recording the elapsed time between service calls followed an exponential distribution having a mean of a call every 24 minutes. In a similar manner, the actual service time of loading or unloading took an average of 8 minutes. If the machine time is valued at Rs 8.50/- per hour, how much does the downtime cost per day? (Assume 8 hour work per day.) — 3 marks

5C. A small railway ticket booking office has two counters – counter 1 for enquiry and counter 2 for ticket booking. Customer arrival is Poisson at 5 per hour to the enquiry and 10 per hour to the ticket booking counter. Exponentially distributed service time in each counter is 4 minutes per customer. Find by how much the average waiting time of a customer in the system reduces at counter 1 (original enquiry counter) when the office decides to go for pooling of resources, i.e., an arriving customer will get enquiry or ticket booking facility at any of the counters. — 4 marks