Reg. No.

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

FIRST SEMESTER M.TECH (ENVIRONMENTAL ENGINEERING) END SEMESTER EXAMINATIONS, NOVEMBER 2017

SUBJECT: COMPUTATIONAL METHODS & OPTIMIZATION TECHNIQUES
[**MAT -5106**]
REVISED CREDIT SYSTEM
(28/11/2017)

Time: 3 Hours   MAX. MARKS: 50

Instructions to Candidates:

> ❖ Answer **ALL** questions.
> ❖ All questions carry equal marks.

1A.	Solve $(x^3 + 1)\, y'' + x^2\, y' - 4xy = 2$, $y(0) = 0$, $y(2) = 4$ with $h = 0.5$.	**4**
1B.	The coefficients a, b, c of the quadratic equation $ax^2 + bx + c = 0$ are determined by throwing a dice three times. Find the probability that (i) the roots are real (ii) the roots are complex.	**3**
1C.	Find the minimum value of $f(x) = x^2 + \dfrac{54}{x}$ by taking $x^{(0)} = 1$, step size 1 using successive quadratic interpolation method.	**3**
2A.	Using Simplex method, solve the following LPP, Maximize $z = 5x_1 + 3x_2$ subject to $x_1 + x_2 \le 2$, $5x_1 + 2x_2 \le 10$, $3x_1 + 8x_2 \le 12$, $x_1, x_2 \ge 0$.	**4**
2B.	The mean inside diameter of a sample of 200 washers produced by a machine is 5.02 mm and the standard deviation is 0.05 mm. The purpose for which these washers are intended allows maximum tolerance in the diameter of 4.96 to 5.08 mm, otherwise the washers are considered to be defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.	**3**

2C.	Two independent random variables X_1 and X_2 have mean values 5, 10 and variance 4, 9. Find covariance between $U = 3X_1 + 4X_2$ and $V = 3X_1 - X_2$.	3
3A.	Using dual simplex method, solve the following LPP, minimize $z = 2x_1 + 2x_2 + 4x_3$ subject to $2x_1 + 3x_2 + 5x_3 \geq 2$, $3x_1 + x_2 + 7x_3 \leq 3$, $x_1 + 4x_2 + 6x_3 \leq 5$, $x_1, x_2, x_3 \geq 0$	4
3B.	Find the real root of the equations $x^2 - y^2 - 4 = 0$ and $x^2 + y^2 - 16 = 0$ near $x_0 = y_0 = 2.828$, by Newton – Raphson's method. Carry out 2 iterations.	3
3C.	The Mendelian theory states that the probabilities of classification a, b,c, d are respectively $\dfrac{9}{16}, \dfrac{3}{16}, \dfrac{3}{16}, \dfrac{1}{16}$. From a sample of 160 the actual numbers observed were 86, 35, 26 and 13. Is this data consistent with the theory at 0.01 significance level ?	3
4A.	Let $\bar{X}$ and S^2 be the mean and variance of a random sample of size 25 from the distribution $N(3,100)$. Evaluate i) $P(0 < \bar{X} < 6)$ ii) $P(55.2 < S^2 < 145.6)$.	4
4B.	Fit the curve $y = ab^x$ to the following data. 	x	2	3	4	5	6	 |---|---|---|---|---|---| | y | 144 | 172.8 | 207.4 | 248.8 | 298.5 |	3
4C.	Minimize the function $f(x) = x^2 - 3x - 20$ in the interval $[0, 5]$ using Fibonacci search method. Take n = 3.	3
5A.	A two dimensional random variable (X, Y) is uniformly distributed over the triangular region R= {(x, y) / 0 < x < y <1}. Find i) P (Y >1) ii) P (X+Y<1/2)	4
5B.	Find the minimum value of $f(x) = x^3 - 3x + 1$ in the interval $[0, 2]$ using Golden section method. Carry out 3 iterations.	3
5C.	Find the median and the quartiles for the following data	3

4B. Fit the curve $y = ab^x$ to the following data.

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

5C. Find the median and the quartiles for the following data

class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
frequency	2	14	27	44	45	18	5	1