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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

A Constituent Institute of Manipal University

I SEMESTER M.TECH (CSE/CSIS) END SEMESTER EXAMINATION, NOV 2017

SUBJECT: Computational Methods and Stochastic Processes (MAT-5108)

(28-11-2017)

Time: 3 Hours

Max. Marks : 50

Instructions to Candidates: Answer all the questions.

Statistical tables will be provided. Missing data may be suitably assumed.

1A. Express the following matrix A as the product of elementary matrices and describe the geometric effect of multiplication by A.

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

1B. A number X is selected from $1, 2, ..., n$. Find a lower bound for

$$P[| X - E(X) | < \sqrt{n^2 - 1}].$$

1C. Use graph theoretic approach to compute the stationary (invariant) probability distribution for Markov chain with three states 1, 2, 3 and transition matrix given by

$$\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Verify that probability distribution you computed is invariant.

(3+3+4)

2A. A computer program while adding numbers rounds each number off to the nearest integer. Suppose that all rounding errors are independent and are uniformly distributed over $(-0.5, 0.5)$. What is the probability that the absolute error in the sum of 1400 numbers is greater than 14?

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2B. With step size $h = \frac{1}{3}$, solve the equation

$$u_{xx} + u_{yy} = 0,$$

$0 \leq x \leq 1, 0 \leq y \leq 1, u(x,1) = u(0,y) = 0, u(1,y) = 18(y - y^2); u(x,0) = 18(x - x^2)$.

2C. Find the coefficient of correlation and the regression equation of y on x given the following data.

X	1	2	3	4	5
Y	2	1	5	3	4

(3+3+4)

3A. Ten children are chosen at random from a population and their heights are found in cm given below:

$$62, 62, 63, 65, 66, 69, 69, 70, 71, 71.$$

Construct two confidence intervals for the population mean by stating the level of significance for each interval.

3B. Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix given by

$$\begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Find the following probabilities assuming that the initial distribution is equally likely for the three states 0, 1 and 2.

(i) $P(X_1 = 1 \mid X_0 = 2)$; (ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.

3C. Find the singular value decomposition(SVD) of the following matrix.

$$\begin{bmatrix} 20 & 4 \\ 10 & 14 \\ 5 & 5 \end{bmatrix}$$

(3+3+4)

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4A. Three persons A, B, C are throwing a ball to each other. A always throws the ball to B and B always to C. However, C is just as likely to throw the ball to B as to A. Find the transition matrix and classify the states.

4B. Let X, Y are two integer valued random variables having the probability function $f(x,y) = k(2x+y)$ where $0 \leq x \leq 2$, $0 \leq y \leq 3$. Find k, $E(X)$, $E(Y/X=2)$.

4C. Use the Simplex method to solve the following linear programming problem.

Maximize $Z = 4x_1 + 3x_2$ subject to

$2x_1 + x_2 \leq 1000$, $x_1 + x_2 \leq 800$, $x_1 \leq 400$, $x_2 \leq 700$,

$x_1 \geq 0, x_2 \geq 0$.

(3+3+4)

5A. Find the mean and variance of the Poisson process. A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a poisson process with mean rate of 1 per week. Suppose that there are 5 spare parts of the component in an inventory and that the supply is not due in next 10 weeks. Find the probability that the machine will not be out of order in the next 10 weeks.

5B. With step size $h = \frac{1}{3}$ solve the Poisson's equation

$$u_{xx} + u_{yy} = -54xy,$$

$0 < x < 1, 0 < y < 1, u(0,y) = u(x,0) = 0, u(1,y) = u(x,1) = 200.$

5C. Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the digit leaving the n^{th} stage of system and X_0 be the digit entering the first stage. At each stage there is a constant probability q that the digit which enters will be transmitted unchanged and probability p otherwise; such that $p + q = 1$. Form a transition probability matrix (t.p.m.) A. Use the diagonalization process to compute A^m and find $P(X_0 = 1 \mid X_m = 1)$ given that $P(X_0 = 0) = a$, $P(X_0 = 1) = 1 - a$.

(3+3+4)

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