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MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

FIRST SEMESTER M.Tech. (DEAC) DEGREE END SEMESTER EXAMINATION
NOV 2017
SUBJECT: DETECTION & ESTIMATION THEORY (ECE – 5104)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates

- Answer **ALL** questions.
- Missing data may be suitably assumed.

1A. In a binary hypothesis problem, the observed random variable under each hypothesis is

$$f_{Y/H_j}(y/H_j) = \frac{1}{\sqrt{2\pi}}\, exp\left(-\frac{(y-m_j)^2}{2}\right), j = 0,1$$

where $m_0 = 0$ and $m_1 = 1$.
(a) Given $P_0 = P_1$, find the decision rule for minimum probability of error.
(b) Find the decision rule for a Neyman-Pearson test if $P_F = 0.005$
(c) Find P_D for the case in (b).
(Note: Use required data from the given values: Q(2.55)=0.0054, Q(2.58)=0.0049, Q(2.60)=0.0046, Q(1.55)=0.0605, Q(1.58)=0.057, Q(1.60)=0.055)

1B. Obtain the expression for minimax rule for binary hypothesis testing. Assume uniform cost assignment.

1C. What do you mean by receiver operating characteristics? What is its significance? Explain with the help of suitable figures.

(5+3+2)

2A. Consider a received signal $Y(t) = \sqrt{(2/T)}\cos(2\pi f_c t) + W(t), 0 < t < T_s$, where $T_s = n/f_c$. Given that T is an unknown positive constatnt and W(t) is AWGN, describe a receiver schematic to estimate $1/\sqrt{T}$.

2B. Consider the composite hypotheses

H_1 : Y = m + N
H_0 : Y = N
where N denotes a Gaussian random variable of zero mean and variance σ^2, and m is unknown. Determine the optimum decision rule given that 'm' is a positive constant. Does UMP test exist for this case?

2C. State and prove the orthogonality principle with respect to linear transformations.

(5+3+2)

3A. Consider a communication system with binary transmission during each duration $T_b = 2\pi/\omega_b$ seconds. The transmitted signal under each hypothesis is

$$H_1: s_1(t) = A\sin\omega_b t\,,\ \ 0 \le t \le T_b$$
$$H_0: s_0(t) = A\sin 2\omega_b t\,,\ \ 0 \le t \le T_b$$

The hypotheses are equally likely. During transmission, the channel superimposes on the signals a white Gaussian noise process with mean zero and power spectral density $N_0/2$. Determine the optimum receiver and calculate the probability of error. Assume minimum probability of error criterion.

3B. The observation sample of the envelope of a received signal is given by the following exponential distribution

$$f_{Y_k}(y_k) = \frac{1}{\theta} exp\left(-\frac{y_k}{\theta}\right), k = 1,2, \dots, K \text{ and } y_k \geq 0$$

θ is an unknown parameter and the observations are statistically independent.
(a) Obtain the ML estimate of θ.
(b) Is the estimator unbiased?
(c) Determine the lower bound on the estimator.

3C. Explain KL series expansion of a random process.

(5+3+2)

4A. Find $\hat{x}_{MMSE}$, the minimum mean-square error estimate, of X from the observation
Y = X + N
Given X is a random variable with density functions

$$f_X(x) = \frac{1}{2}[\delta(x-1) + \delta(x+1)]$$

and N is a Gaussian random variable with zero mean and variance σ^2.

4B. The observation Y is given by Y = X + N, where X and N are two random variables. N is Gaussian distributed with mean one and variance σ^2, and X is uniformly distributed over the interval [0, 2]. Determine the MAP estimate of the parameter X.

4C. What do you mean by Cramer Rao Lower Bound? Explain.

(5+3+2)

5A. (i) Find a set of orthonormal basis functions for the set of signals shown in Figure Q.5.A .

(ii) Find the vector corresponding to each signal for the orthonormal basis set found in (i), and sketch the signal constellation.

5B. Derive the expression for the frequency response of an optimum unrealizable Weiner filter.

5C. Let $Y_1 \dots Y_K$ be K observations of a random variable Y. The observations Y_k are iid random variables with parameter θ.

$$f_{Y_k/\Theta}\left({y_k}/{\theta}\right) = \begin{cases} \theta\, e^{-\theta y_k} & y_k \geq 0 \\ 0 & y_k < 0 \end{cases}$$

Find the ML estimate of θ.

(5+3+2)

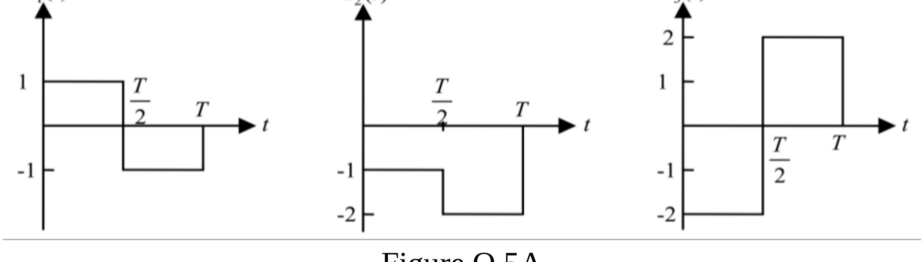

Figure Q.5A