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MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

FIRST SEMESTER M.TECH (SOFTWARE ENGINEERING) DEGREE END SEMESTER EXAMINATION-NOVEMBER 2017
SUBJECT: MATHEMATICAL LOGIC (ICT 5122)
(REVISED CREDIT SYSTEM)

TIME: 3 HOURS 18/11/2017 MAX. MARKS: 50

Instructions to candidates

- Answer **ALL** questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Consider the argument "*When Alexis attends math class, her friends Guppy and Desmorelda also attend. Since Desmorelda is in love with Luke, Lukes attendance at class is a sufficient condition for her to attend as well. On the other hand, for Desmorelda to attend class it is necessary that Alexis also be there (as she needs someone to talk to during the boring portions of the class). Therefore, Luke wont attend class unless Guppy also attends*". Consider first letter in lower case for each name, and write the sequent in propositional logic. Prove the sequent using natural deduction rules. [5]

1B. Use soundness theorem to check the validity of the following sequents:

i) $\neg p \vee (q \rightarrow p) \vdash \neg p \wedge q$

ii) $\neg r \rightarrow (p \vee q), r \wedge \neg q \vdash r \rightarrow q$

iii) $p \rightarrow (q \rightarrow r) \vdash p \rightarrow (r \rightarrow q)$.

[3]

1C. Prove the validity of the following sequent.

$$\vdash (p \rightarrow q) \rightarrow ((r \rightarrow s) \rightarrow (p \wedge r \rightarrow q \wedge s))$$

[2]

2A. Translate the following sentences into well formed formulas in predicate logic:

i) Anyone who is persistent can learn logic

ii) If anyone can solve the problem, Sam can

iii) Nobody in the statistics class is smarter than everyone in the logic class

iv) Any sets that have the same members are equal

v) Nobody loves a loser

[5]

2B. Prove the equivalence $\forall x \phi \wedge \forall x \psi \dashv\vdash \forall x(\phi \wedge \psi)$. [3]

2C. Let $\mathcal{F} \stackrel{def}{=} \{i\}$ and $\mathcal{P} \stackrel{def}{=} \{R, F\}$; where i is a constant, F a predicate symbol with arity one and R a pedicate symbol with arity two. A model $\mathcal{M}$ contains a set of concrete elements A-which may be a set of states of a computer program. The interpretations $i^{\mathcal{M}}$, $R^{\mathcal{M}}$, and $F^{\mathcal{M}}$ may then be a designated initial state, a state transition relation, and a set of final (accepting) states, respectively. Let $A \stackrel{def}{=} \{a, b, c\}$, $i^{\mathcal{M}} \stackrel{def}{=} a$, $R^{\mathcal{M}} \stackrel{def}{=} \{(a,a),(a,b),(a,c),(b,c),(c,c)\}$, and $F^{\mathcal{M}} \stackrel{def}{=} \{b, c\}$. For the given model $\mathcal{M}$, check the satisfaction relation $\mathcal{M} \vDash \phi$, where ϕ :

i) $\forall x \forall y \forall z (R(x,y) \wedge R(x,z) \to y = z)$

ii) $\forall x \exists y R(x,y)$ [2]

3A. Write Backus Naur Form (BNF) for CTL, and define its semantic satisfaction relation, $\mathcal{M}, s \vDash \phi$, over a model $\mathcal{M} = (S, \to, L)$ by structural induction on ϕ. [5]

3B. Consider the Kripke model $\mathcal{M}$ of Figure Q.3B. Indicate for each of the following LTL formulae the set of states for which these formulae hold the relation $\mathcal{M}, \vDash \phi$.

i) Xa

ii) $XXXa$

iii) Gb

iv) GFa

v) $G(b\ U\ a)$

vi) $F(a\ U\ b)$ [3]

3C. Express the following properties in CTL, and LTL whenever possible. If neither is possible, try to express the property in CTL^*:

i) Whenever p is followed by q (after finitely many steps), then the system enters an 'interval' in which no r occurs until t.

ii) Property p is true for every second state along a path. [2]

4A. Formalize the wise-men puzzle in the modal logic $KT45^n$. [5]

4B. Prove the first entailment of wise-men puzzle obtained from Q.4A. [3]

4C. Consider the Kripke model $\mathcal{M}$ depicted in Figure Q.4C. Find for each of the following a world which satisfies it:

i) $\Box\neg p \wedge \Box\Box\neg p$

ii) $\Diamond(p \vee \Diamond q)$ [2]

5A. Describe different *types* that are recognized by NuSMV. [5]

5B. Write NuSMV syntax for the following in reference to a finite state machine:

i) DEFINE Declaration

ii) Array Define Declaration

iii) CONSTANTS Declarations [3]

5C. Write syntax for CTL formulas recognized by the NuSMV. [2]

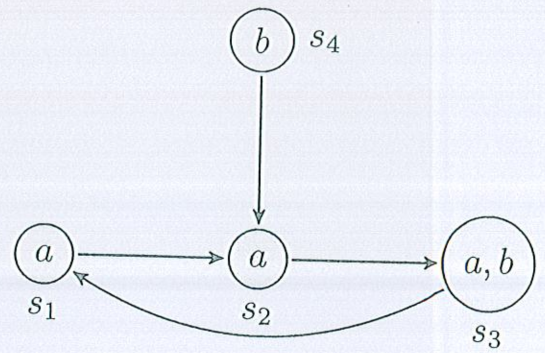

Figure: Q.3B

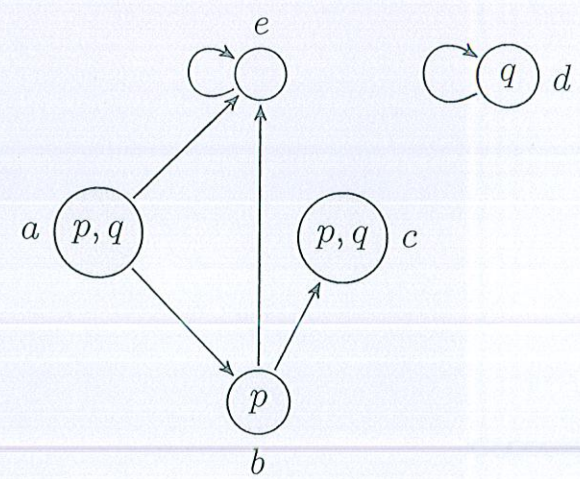

Figure: Q.4C