Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

I SEMESTER M.TECH (ICT) END SEMESTER MAKE-UP EXAMINATION, JAN 2018

SUBJECT: PROBABILITY AND STOCHASTIC PROCESS (MAT-5111)

(02-01-2018)

Time: 3 Hours Max. Marks : 50

Answer all the questions. Statistical tables will be provided.

Missing data may be suitably assumed.

1A. Let X be a random variable having p.d.f. $f(x) = \dfrac{1}{1+x^2}$ for $-\infty < x < \infty$. Then find the p.d.f. of $Y = \dfrac{1}{X}$

1B. From a normally distributed products in a manufacturing company 7% of the items have their values less than 35 and 89% items have their values less than 63. Find the mean and standard deviation of the distribution.

1C. Solve the game whose payoff matrix is given by Player A $\begin{pmatrix} \text{Player B} & & \\ & B_1 & B_2 \\ A_1 & 5 & 1 \\ A_2 & 3 & 4 \end{pmatrix}$

(4+3+3)

2A. The pdf of one dimensional continuous random variable X is given by

$$f(x,y) = \begin{cases} x, & \text{if } 0 \leq x \leq 1 \\ 2-x, & 1 < x < 2 \\ 0, & \text{else where} \end{cases}$$

Compute (i) the cumulative density function (c.d.f.) of X (ii) $P(X \geq \dfrac{3}{2})$

2B. In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

2C. Solve the game by graphical method

$$\text{Player A} \begin{pmatrix} \text{Player B} & & & & \\ & B_1 & B_2 & B_3 & B_4 \\ A_1 & 2 & 1 & 0 & -2 \\ A_2 & 1 & 0 & 3 & 2 \end{pmatrix}$$

(4+3+3)

3A. Consider a random experiment that consists of two throws of a fair die. Let X be the number of 4's and Y be the number of 5's obtained in the 2 throws. Then, (i) find the joint probability distribution of (X, Y) (ii) Compute $P(2X + Y < 3)$

3B. Find the fixed probability vector of the regular stochastic matrix

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

3C. A T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in. If the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day. What is the repairman's expected idle time each day? How many jobs are ahead of the avearge set just brought in?

(4+3+3)

4A. Let $\{X_n; n \geq 0\}$ be a Markov Chain with three states 0,1,2 and transition probability matrix given by

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \end{pmatrix}$$

Assume that $P\{X_0 = i\} = \dfrac{1}{3}$ for $i = 0, 1, 2$

(i) Compute the two step transition matrix (ii)$P\left(X_2 = 1 | X_0 = 0\right)$ (iii) $P\left(X_2 = 2\right)$ in $P^{(2)}$

4B. Let X be a continuous random variable with p.d.f. $f(x) := \begin{cases} me^{-mx}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$ where m is a positive parameter. Then compute the m.g.f. of X. Hence find $E(X)$ and $E(2X + 3)$.

4C. Let $(X_1, X_2, ..., X_n)$ be a a random sample of size n from a population X having distribution $f\left(x, \theta\right) = \theta x^{\theta - 1}$ for $0 < x < 1$ and $\theta > 0$. Find the M.L.E. of θ.

(4+3+3)

5A. Let $X_1, X_2, ..., X_{25}$ and $Y_1, Y_2, ..., Y_{25}$ be two independent samples from a normal population $N(3, 16)$ and $N(4, 9)$ respectively. Using central limit theorem, find $P(\bar{X} - \bar{Y} > 0)$.

5B.A random sample of size 17 from a normal population is found to have $\bar{X} = 4.7$ and $s^2 = 5.76$. Find a 95% confidence interval for the population mean μ.

5C. Consider a 3-state Markov chain having transition probability matrix $P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ then, (i) draw the stochastic graph and (ii) Test whether the Markov chain is ergodic or not.

(4+3+3)