Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

I SEMESTER M.TECH (ICT) END SEMESTER EXAMINATION, NOV 2017
SUBJECT: PROBABILITY AND STOCHASTIC PROCESS (MAT-5111)
(28-11-2017)

Time: 3 Hours Max. Marks : 50

Answer all the questions. Statistical tables will be provided.

1A. Let $\{X_n; n \geq 0\}$ be a Markov Chain with three states 0,1,2 and transition probability matrix given by

$$P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/2 & 1/4 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

Find the following probabilities assuming that the initial distribution is equally likely for the three states 0,1,2

(i)$P\left(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\right)$ (ii) $P\left(X_2 = 1 | X_1 = 1\right)$

1B. Obtain the moment generating function (m.g.f.) of a Binomial distribution.

1C. The mean of a random sample of size 25 from a normal population $N\left(\mu, 4\right)$ was found to be 78.3. Find a 99% confidence interval for μ.

(4+3+3)

2A. Let $\bar{X}$ be the mean of a random sample of size 15 from a population X having p.d.f. $f(x) := \begin{cases} 3x^2, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$ Using central limit theorem, find $P\left(\frac{3}{5} < \bar{X} < \frac{4}{5}\right)$.

2B. The annual rainfall at a certain locality is known to be normally distributed random variable with mean 29.5 inches and standard deviation 2.5 inches. How many inches of rain (annually) is exceeded about 5 percent of the time.

2C. Define a two person zero sum game. Solve the game whose payoff matrix is given by

Player A / Player B

	B_1	B_2
A_1	6	-3
A_2	-3	0

(4+3+3)

3A. If X is a continuous random variable with the p.d.f. $f(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$ then, find $E(X)$ and $V(X)$.

3B. Let $(X_1, X_2, ..., X_n)$ be a a random sample of size n from a population X having distribution $f(x, \theta) = \frac{\theta^x e^{-\theta}}{x!}$ for $\theta > 0$. Find the M.L.E. of θ.

3C. Consider a 3-state Markov chain having transition probability matrix $P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \end{pmatrix}$ then, (i) draw the stochastic graph and (ii) classify the nature of states.

(4+3+3)

4A. Let X be a random variables which follows a uniform distribution over $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Then find the p.d.f. of $Y = \tan X$.

4B. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blades to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective and two defective blades respectively in a consignment of 20, 000 packets.

4C. Find the fixed probability vector of the regular stochastic matrix

$$P = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$$

(4+3+3)

5A. The joint pdf of 2 dimensional discrete random variable (X, Y) is given by

$$f(x, y) = \begin{cases} c(2x + y), & \text{if } x = 0, 1, 2 \ ; \ y = 0, 1, 2, 3 \\ 0, & \text{else where} \end{cases}$$

Compute (i)the value of c (ii) $E(X), E(XY)$ (iii)$P(X + Y > 1)$

5B. Solve the game by dominance rule

Player A \ Player B	B_1	B_2	B_3	B_4
A_1	5	−10	9	0
A_2	6	7	8	1
A_3	8	7	15	1
A_4	3	4	−1	4

5C. In a supermarket, the average arrival rate of customer is 10 every 30 minutes, following Poisson process. The average time taken by a cashier to list and calculate the customer's purchase is 2.5 minutes, following exponential distribution. What is the probability that the queue length exceeds 6? What is the expected time spent by a customer in the system?

(4+3+3)