Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

FIRST SEMESTER M.TECH.

(CONTROL SYSTEMS AND ASTRONOMY AND SPACE ENGINEERING)

END SEMESTER EXAMINATION , DECEMBER 2017

SUBJECT: APPLIED LINEAR ALGEBRA AND PROBABILITY (MAT – 5110)

REVISED CREDIT SYSTEM

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** questions.
- Missing data may be suitable assumed.

1A.	Following table gives the amount, in milligrams(mg), of vitamin A, vitamin C and calcium contained in 1 gram of four different foods. For example, food 1 has 10mg of vitamin A, 50 mg of vitamin C, and 60 mg of calcium per gram of food. Suppose that a dietician want to prepare a meal that provides 200 mg of vitamin A, 250 mg of vitamin C, and 300 mg of calcium. How much of each food should be used ?	**3**

	Food 1	Food 2	Food 3	Food 4
Vitamin A	10	30	20	10
Vitamin C	50	30	25	10
Calcium	60	20	40	25

1B.	Are these matrices $\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}$, $\begin{bmatrix}-1 & 2\\ 3 & 2\end{bmatrix}$ and $\begin{bmatrix}5 & -6\\ -3 & -2\end{bmatrix}$ linearly independent ?	**3**
1C.	Let T : $R^2 \to R^2$ be the transformation that performs a rotation by 45^0, followed by a reflection through the origin. (i) Find the matrix of T relative to the standard basis. (ii) Apply the transformation to the square with vertices (0, 0), (1,0), (1,1) and (0,1) and give a sketch of the result.	**4**
2A.	The W is subspace of an inner product space V then show that (i) W is subspace of V (ii) $W \cap W^{\perp}=\{0\}$.	**3**
2B.	For $S = \left\{ \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}-1\\3\\-1\end{bmatrix}, \begin{bmatrix}1\\2\\-1\end{bmatrix}, \begin{bmatrix}0\\6\\1\end{bmatrix}, \begin{bmatrix}-3\\4\\5\end{bmatrix} \right\}$ find span (S).	**3**
2C.	Describe the conic section C whose equation is $27x^2 - 18xy + 3y^2 + x + 3y = 0$	**4**

3A.	If $S = \{v_1, v_2, \ldots\ldots v_n\}$ is an orthogonal set of nonzero vectors in an inner product space V, then show that S is linearly independent.	**3**
3B.	Give the definition of inner product on the vector space V and state Cauchy - Schwartz inequality also Triangle inequality.	**3**
3C.	Suppose that two brine storage tanks are connected with two pipes used to exchange solutions between them. The first pipe allows water from tank 1 to enter tank 2 at a rate of 5 gal/min. The second pipe reverses the process allowing water to flow from tank 2 to tank 1, also at a rate of 5 gal/min. Initially, the first tank contains a well – mixed solution of 8 lb of salt in 50 gal. of water, while the second tank contains 100 gal. of pure water. (i) Find the linear system of differential equations to describe the amount of salt in each tank at time t. (ii) Solve the system of equations by reducing it to an uncoupled system.	**4**
4A	For the basis sets $B_1 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$ and let $B_2 = \left\{ \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$ Find the transition matrix from B_1 to B_2 .	**3**
4B	Is $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ diagonalizable ? Justify your answer.	**3**
4C	Let $V = P_2$ with inner product defined by $\langle p, q \rangle = \int_{-1}^{1} p(x)\, q(x)\, dx$. Let p(x) = x and q(x) = x^2 the find projection of p over q	**4**
5A	An urn contains 10 black and 10 white balls. Find the probability of drawing two balls of the same colour.	**3**
5B	If $A = \begin{bmatrix} 1 & 3 \\ 1 & 3 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$. (i) Find the least squares solution to Ax = B (ii) Find the orthogonal projection of b onto W = col(A) and the decomposition of the vector $b = w_1 + w_2$, where w_1 is in W and w_2 is in $W^{\perp}$.	**3**
5C	State and prove Bayes theorem on probability.	**4**

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